

mat 1 - lista 2

$$① a) 3^x = \frac{1}{81}$$

$$3^x = \frac{1}{3^4} = 3^{-4}$$

$$x = -4$$

$$S = \{-4\}$$

$$b) 10^{1-4x} = 0,001$$

$$10^{1-4x} = 10^{-3}$$

$$1-4x = -3$$

$$4x = 4$$

$$x = 1 \Rightarrow S = \{1\}$$

$$c) \left(\frac{5}{3}\right)^{2x} = \frac{27}{125}$$

$$\left(\frac{5}{3}\right)^{2x} = \left(\frac{3}{5}\right)^3 = \left(\frac{5}{3}\right)^{-3}$$

$$2x = -3$$

$$x = -\frac{3}{2}$$

$$S = \{-\frac{3}{2}\}$$

$$d) 10^x = \sqrt{0,1}$$

$$10^x = \sqrt{10^{-1}}$$

$$10^x = (10^{-1})^{1/2}$$

$$10^x = 10^{-1/2}$$

$$x = -1/2$$

$$S = \{-1/2\}$$

$$e) 2^{3x+1} = 4^{x-2}$$

$$2^{3x+1} = (2^2)^{x-2}$$

$$2^{3x+1} = 2^{2x-4}$$

$$3x+1 = 2x-4$$

$$x = -5$$

$$S = \{-5\}$$

$$f) 125^{x+1} = \frac{1}{\sqrt[3]{625}} = \frac{1}{\sqrt[3]{5^4}} = \frac{1}{5^{4/3}}$$

$$(5^3)^{x+1} = 5^{-4/3}$$

$$3x+3 = -4/3$$

$$3x = -13/3$$

$$x = -\frac{13}{9} \Rightarrow S = \{-13/9\}$$

$$g) 81^{x+2} = 1$$

$$(3^4)^{x+2} = 3^0$$

$$4x+8 = 0$$

$$4x = -8$$

$$x = -2$$

$$S = \{-2\}$$

$$h) 2^{1+x} + \sqrt{8} = \sqrt{72}$$

$$2^{1+x} + 2\sqrt{2} = 6\sqrt{2}$$

$$2^{1+x} = 4\sqrt{2} = 2^2 \cdot 2^{1/2} = 2^{5/2}$$

$$2^{1+x} = 2^{5/2}$$

$$1+x = 5/2$$

$$x = \frac{5}{2} - 1$$

$$x = 3/2 \Rightarrow S = \{3/2\}$$

$$i) 3^{x^2-x-3} = 27$$

$$3^{x^2-x-3} = 3^3$$

$$x^2 - x - 3 = 3$$

$$x^2 - x - 6 = 0$$

$$(x+2)(x-3) = 0$$

$$x = -2 \text{ ou } x = 3$$

$$S = \{-2, 3\}$$

$$j) \frac{\sqrt{3} \cdot \sqrt[4]{9}}{3} = 3^x$$

$$\frac{3^{1/2} \cdot 3^{2/4} = 3^{1/2}}{3} = 3^x$$

$$\frac{3^1}{3^1} = 3^x$$

$$3^0 = 3^x$$

$$x = 0 \Rightarrow S = \{0\}$$

$$k) 4^x = \sqrt[3]{32}$$

$$(2^2)^x = (2^5)^{1/3}$$

$$2^{2x} = 2^{5/3}$$

$$2x = 5/3$$

$$x = 5/6 \Rightarrow S = \{5/6\}$$

$$\textcircled{2} \quad a) \quad 2^{x-1} + 2^x + 2^{x+1} - 2^{x+2} + 2^{x+3} = 120$$

$$2^x \cdot 2^{-1} + 2^x + 2^x \cdot 2^1 - 2^x \cdot 2^2 + 2^x \cdot 2^3 = 120$$

$$\boxed{2^x = y}$$

$$\frac{y}{2} + y + 2y - 4y + 8y = 120$$

$$7,5y = 120$$

$$y = 16 \Rightarrow 2^x = 16$$

$$2^x = 2^4$$

$$x = 4 \Rightarrow S = \{4\}$$

$$b) \quad 5^{x-2} - 5^x + 5^{x+1} = 505$$

$$5^x \cdot 5^{-2} - 5^x + 5^x \cdot 5^1 = 505$$

$$\boxed{5^x = y}$$

$$\frac{y}{25} - y + 5y = 505$$

$$4y + \frac{y}{25} = 505$$

$$\frac{101y}{25} = \frac{505}{1} \Rightarrow y = 125$$

$$y = 125 \Rightarrow 5^x = 125$$

$$5^x = 5^3$$

$$x = 3 \Rightarrow S = \{3\}$$

$$c) 3^{x-1} - 3^x + 3^{x+1} + 3^{x+2} = 306$$

$$3^x \cdot 3^{-1} - 3^x + 3^x \cdot 3^1 + 3^x \cdot 3^2 = 306$$

$$\boxed{3^x = y}$$

$$\frac{y}{3} - y + 3y + 9y = 306$$

$$11y + \frac{y}{3} = 306$$

$$1 \frac{34y}{3} = \cancel{306}^9$$

$$y = 27 \Rightarrow 3^x = 27$$

$$3^x = 3^3$$

$$x = 3 \Rightarrow S = 137$$

$$d) 4^{x+2} + 4^{x-1} - 4^{x+1} + 4^x = 212$$

$$4^x \cdot 4^2 + 4^x \cdot 4^{-1} - 4^x \cdot 4^1 + 4^x = 212$$

$$\boxed{4^x = y}$$

$$16y + \frac{y}{4} - 4y + y = 212$$

$$13y + \frac{y}{4} = 212$$

$$1 \frac{53y}{4} = \cancel{212}^3$$

$$y = 16 \Rightarrow 4^x = 16$$

$$4^x = 4^2$$

$$x = 2 \Rightarrow S = 127$$

$$2) \quad 5^{x-1} + 5^x + 5^{x+1} = 775$$

$$5^x \cdot 5^{-1} + 5^x + 5^x \cdot 5^1 = 775$$

$$\frac{y}{5} + y + 5y = 775$$

$$6y + \frac{y}{5} = 775$$

$$\frac{31y}{5} = 775$$

$$y = 125 \Rightarrow 5^x = 125$$

$$5^x = 5^3$$

$$x = 3 \Rightarrow S = \{3\}$$

$$1) \quad 4^{2x-2} - 17 \cdot 4^{x-2} + 1 = 0$$

$$4^{2x} \cdot 4^{-2} - 17 \cdot 4^x \cdot 4^{-2} + 1 = 0$$

$$\frac{y^2}{16} - \frac{17y}{16} + 1 = 0$$

$$y^2 - 17y + 16 = 0$$

$$(y-1)(y-16) = 0$$

$$y=1 \quad \text{ou} \quad y=16$$

$$\Downarrow$$

$$4^x = 1$$

$$4^x = 4^0$$

$$x = 0$$

$$\Downarrow$$

$$4^x = 16$$

$$4^x = 4^2$$

$$x = 2$$

$$S = \{0; 2\}$$

$$\boxed{5^x = y}$$

$$\boxed{\begin{array}{l} 4^x = y \\ 4^{2x} = y^2 \end{array}}$$

$$g) 3^{2x} - 4 \cdot 3^x + 3 = 0$$

$$(3^x)^2 - 4 \cdot 3^x + 3 = 0$$

$$y^2 - 4y + 3 = 0$$

$$(y-3)(y-1) = 0$$

$$y = 3 \text{ ou } y = 1$$

$$\Downarrow$$

$$3^x = 3^1$$

$$x = 1$$

$$\Downarrow$$

$$3^x = 1$$

$$3^x = 3^0$$

$$x = 0$$

$$\boxed{3^x = y}$$

$$S = \{0; 1\}$$

$$h) 4^x - 5 \cdot 2^x + 4 = 0$$

$$(2^2)^x - 5 \cdot 2^x + 4 = 0$$

$$(2^x)^2 - 5 \cdot 2^x + 4 = 0$$

$$y^2 - 5y + 4 = 0$$

$$(y-4)(y-1) = 0$$

$$y = 4 \text{ ou } y = 1$$

$$\Downarrow$$

$$2^x = 4$$

$$2^x = 2^2$$

$$x = 2$$

$$\Downarrow$$

$$2^x = 1$$

$$2^x = 2^0$$

$$x = 0$$

$$\boxed{2^x = y}$$

$$S = \{0; 2\}$$

$$i) 5^x - 5^{2-x} = 24$$

$$5^x - 5^2 \cdot 5^{-x} = 24$$

$$y - \frac{25}{y} = 24 \quad (y \neq 0)$$

$$y^2 - 25 = 24y$$

$$y^2 - 24y - 25 = 0$$

$$(y - 25)(y + 1) = 0$$

$$y = 25 \text{ or } y = -1$$

$$\Downarrow$$

$$5^x = 25$$

$$5^x = 5^2$$

$$x = 2$$

$$\Downarrow$$

$$\cancel{5^x = -1}$$

$$\boxed{5^x = y}$$

$$S = \{2\}$$

$$j) 2^{2x+1} - 3 \cdot 2^{x+2} = 32$$

$$2^{2x} \cdot 2^1 - 3 \cdot 2^x \cdot 2^2 = 32$$

$$(2^x)^2 \cdot 2 - 12 \cdot 2^x = 32$$

$$2y^2 - 12y = 32$$

$$y^2 - 6y - 16 = 0$$

$$(y + 2)(y - 8) = 0$$

$$y = -2 \text{ or } y = 8$$

$$\Downarrow$$

$$\cancel{2^x = -2}$$

$$\Downarrow$$

$$2^x = 8$$

$$2^x = 2^3$$

$$x = 3$$

$$\boxed{2^x = y}$$

$$S = \{3\}$$

$$k) \quad 5^{2x-1} - 10 \cdot 5^{x-1} - 75 = 0$$

$$(5^{2x} \cdot 5^{-1} - 10 \cdot 5^x \cdot 5^{-1} - 75 = 0$$

$$\boxed{5^x = y}$$

$$\frac{(5^x)^2}{5} - \frac{10}{5} \cdot 5^x - 75 = 0$$

$$\frac{y^2}{5} - 2y - 75 = 0$$

$$y^2 - 10y - 375 = 0$$

$$(y + 15)(y - 25) = 0$$

$$y = -15 \quad \text{ou} \quad y = 25$$

$$\Downarrow$$

$$\cancel{5^x = -15}$$

$$\Downarrow$$

$$5^x = 25$$

$$5^x = 5^2$$

$$x = 2$$

$$S = \{2\}$$

③

$$\frac{4^{x/2}}{2} - \frac{2^{x-1}}{3} = \frac{4}{3}$$

$$\frac{(2^2)^{x/2}}{2} - \frac{2^x \cdot 2^{-1}}{3} = \frac{4}{3}$$

$$\frac{2^x}{2} - \frac{2^x}{6} = \frac{4}{3} \quad (\times 6)$$

$$3 \cdot 2^x - 2^x = 2 \cdot 4$$

$$2 \cdot 2^x = 8$$

$$2^x = 4 \Rightarrow 2^x = 2^2 \Rightarrow x = 2$$

$$(4) \quad 9^{x-\frac{1}{2}} - \frac{4}{3^{1-x}} = -1$$

$$\boxed{3^x = y}$$

$$(3^2)^{x-\frac{1}{2}} - \frac{4}{3^1 \cdot 3^{-x}} = -1$$

$$3^{2x-1} - \frac{4 \cdot 3^x}{3} = -1$$

$$\frac{(3^x)^2}{3} - \frac{4}{3} \cdot 3^x + 1 = 0$$

$$\frac{y^2}{3} - \frac{4}{3} y + 1 = 0$$

$$y^2 - 4y + 3 = 0$$

$$(y-3)(y-1) = 0$$

$$y = 3 \text{ ou } y = 1$$

$$\Downarrow$$

$$3^x = 3$$

$$3^x = 3^1$$

$$x = 1$$

$$\Downarrow$$

$$3^x = 1$$

$$3^x = 3^0$$

$$x = 0$$

$$S = \{0; 1\}$$

$$⑤ a) 7^x = 5^x$$

$$\frac{7^x}{5^x} = 1$$

$$\left(\frac{7}{5}\right)^x = 1$$

$$\left(\frac{7}{5}\right)^x = \left(\frac{7}{5}\right)^0$$

$$x = 0$$

$$S = \{0\}$$

$$b) \sqrt{5^{x-2}} \cdot \sqrt[2]{25^{2x-5}} - 2\sqrt{5^{3x-2}} = 0 \quad \leadsto x > 0$$

$$(5^{x-2})^{1/2} \cdot [(5^2)^{2x-5}]^{1/2} - (5^{3x-2})^{1/2} = 0$$

$$5^{\frac{x-2}{2}} \cdot 5^{\frac{4x-10}{x}} - 5^{\frac{3x-2}{2x}} = 0$$

✓

$$5^{\frac{x^2+6x-20}{2x}} = 5^{\frac{3x-2}{2x}}$$

$$\frac{x-2}{2/x} + \frac{4x-10}{x/2}$$

$$\frac{x^2 - 2x + 8x - 20}{2x}$$

$$\frac{x^2 + 6x - 20}{2x}$$

$$\frac{x^2 + 6x - 20}{2x} = \frac{3x-2}{2x}$$

$$x \neq 0$$

$$x^2 + 6x - 20 - 3x + 2 = 0$$

$$x^2 + 3x - 18 = 0$$

$$(x-3)(x+6) = 0$$

$$x = 3 \text{ ou } x = \cancel{6}$$

$$S = \{3\}$$

$$c) \quad 4^x - 3^{x-\frac{1}{2}} = 3^{x+\frac{1}{2}} - 2 \quad \xrightarrow{2x-1}$$

$$4^x - 3^x \cdot 3^{-1/2} = 3^x \cdot 3^{1/2} - 4^x \cdot 4^{-1/2} \quad (\div 3^x)$$

$$\frac{4^x}{3^x} - 3^{-1/2} = 3^{1/2} - \frac{4^x}{3^x} \cdot 4^{-1/2}$$

$$\boxed{\left(\frac{4}{3}\right)^x = y}$$

$$y - \frac{1}{\sqrt{3}} = \sqrt{3} - y \cdot \frac{1}{\sqrt{4}}$$

$$y - \frac{1}{\sqrt{3}} = \sqrt{3} - \frac{y}{2}$$

$$y + \frac{y}{2} = \sqrt{3} + \frac{1}{\sqrt{3}}$$

$$\frac{3y}{2} = \frac{4}{\sqrt{3}}$$

$$\frac{9y^2}{4} = \frac{16}{3}$$

$$y^2 = \frac{4^3}{3^3}$$

$$y = \sqrt{\left[\left(\frac{4}{3}\right)^3\right]^{1/2}}$$

$\Downarrow$

$$\left(\frac{4}{3}\right)^x = \left[\left(\frac{4}{3}\right)^3\right]^{1/2}$$

$$x = 3/2$$

$$S = \{3/2\}$$

6

$$N(t) = 1200 \cdot 2^{0,4t}$$

$$38400 = 1200 \cdot 2^{0,4t}$$

$$32 = 2^{0,4t}$$

$$2^5 = 2^{0,4t}$$

$$0,4t = 5$$

$$t = 12,5 \text{ horas}$$

7

a) $f(t) = g(t)$

$$2^{t+2} + 75 = 2^{t+1} + 139$$

$$2^{t+2} - 2^{t+1} - 64 = 0$$

$$2^t \cdot 2^2 - 2^t \cdot 2^1 - 64 = 0$$

$$\boxed{2^t = y}$$

$$4y - 2y - 64 = 0$$

$$2y = 64$$

$$y = 32$$

$\Downarrow$

$$2^t = 32$$

$$2^t = 2^5$$

$$t = 5 \text{ años}$$

b) $f(t) = 2^{t+2} + 75$

$$f(7) = 2^{7+2} + 75$$

$$f(7) = 2^9 + 75$$

$$f(7) = 512 + 75$$

$$f(7) = 587 \text{ individuos}$$

8

$$Y = \frac{Y_0}{4} \quad \text{e} \quad Y = Y_0 \cdot 2^{-0,5t}$$

$$\hookrightarrow Y_0 = 4Y$$

$$Y = 4Y \cdot 2^{-0,5t}$$

$$\frac{1}{4} = 2^{-0,5t}$$

$$2^{-2} = 2^{-0,5t}$$

$$-0,5t = -2$$

$$t = 4 \text{ horas}$$

9

$$m(t) = 100 \cdot 2^{t/3}$$

$$51200 = 100 \cdot 2^{t/3}$$

$$512 = 2^{t/3}$$

$$2^9 = 2^{t/3}$$

$$\frac{t}{3} = 9$$

$$t = 27 \text{ horas}$$

$\Downarrow$

$$24 \text{ h} + 3 \text{ h}$$

$\Downarrow$

$$1 \text{ dia e } 3 \text{ horas}$$

(A)