

5ª parte

$$\textcircled{1} \text{ a) } 2^x = 3$$

$$\log_2 3 = x$$

$$x = \frac{\log 3}{\log 2}$$

$$x = \frac{0,48}{0,30}$$

$$x = \frac{48}{30} = \frac{8}{5}$$

$$\text{b) } 2^x = 5$$

$$\log_2 5 = x$$

$$x = \frac{\log 5}{\log 2}$$

$$x = \frac{\log \frac{10}{2}}{\log 2}$$

$$x = \frac{\log 10 - \log 2}{\log 2}$$

$$x = \frac{1 - 0,3}{0,3} = \frac{0,7}{0,3} = \frac{7}{3}$$

$$\text{c) } 2^x = 12$$

$$\log_2 12 = x$$

$$x = \frac{\log 2^2 \cdot 3}{\log 2}$$

$$x = \frac{2 \log 2 + \log 3}{\log 2}$$

$$x = \frac{2 \cdot 0,3 + 0,48}{0,3}$$

$$x = \frac{1,08}{0,3} = \frac{108}{30} = \frac{18}{5}$$

$$\text{d) } 3 \cdot 2^x = 10$$

$$2^x = \frac{10}{3}$$

$$x = \log_2 10/3$$

$$x = \frac{\log 10 - \log 3}{\log 2}$$

$$x = \frac{1 - 0,48}{0,3}$$

$$x = \frac{0,52}{0,30} = \frac{52}{30}$$

$$x = \frac{26}{15}$$

$$\text{e) } (0,3)^x = 1,5$$

$$x = \log_{0,3} 1,5$$

$$x = \frac{\log 3/2}{\log 3/10}$$

$$x = \frac{\log 3 - \log 2}{\log 3 - \log 10}$$

$$x = \frac{0,48 - 0,30}{0,48 - 1}$$

$$x = \frac{0,18}{-0,52} = -\frac{18}{52}$$

$$x = -\frac{9}{26}$$

$$\text{f) } 2^{2x} - 8 \cdot 2^x + 12 = 0$$

$$(2^x)^2 - 8 \cdot 2^x + 12 = 0$$

$$y^2 - 8y + 12 = 0$$

$$(y - 2)(y - 6) = 0$$

$$y = 2 \quad y = 6$$

$$2^x = 2 \quad 2^x = 6$$

$$x = 1 \quad x = \log_2 6$$

$$x = \frac{\log 2 + \log 3}{\log 2}$$

$$x = \frac{0,3 + 0,48}{0,3}$$

$$x = \frac{0,78}{0,30} = \frac{78}{30}$$

$$x = \frac{13}{5}$$

(2) $P = P_0(0,9)^n$
 $0,3 = 1(0,9)^n$
 $\log_{0,9} 0,3 = n$

$\rightarrow n = \frac{\log 3/10}{\log 9/10}$
 $\rightarrow n = \frac{0,48 - 1}{2,048 - 1}$
 $n = \frac{-0,52}{-0,04} = \frac{52}{4}$
 $n = 13$ (E)

(3) $m(t) = m_0 \cdot 10^{-t/70}$
 $\frac{1}{8} m_0 = m_0 \cdot 10^{-t/70}$
 $\frac{1}{8} = 10^{-t/70}$
 $\log_{10} 1/8 = -t/70$

$\rightarrow -\frac{t}{70} = \log 2^{-3}$
 $\rightarrow -\frac{t}{70} = -3 \cdot \log 2$
 $\rightarrow -\frac{t}{70} = -3 \cdot 0,3$
 $\rightarrow -\frac{t}{70} = -0,9$
 $t = 63$ anos

(4) $P = P_0 \cdot e^{-t/250}$
 $\frac{1}{4} P_0 = P_0 \cdot e^{-t/250}$
 $\frac{1}{4} = e^{-t/250}$
 $\log_e 1/4 = -\frac{t}{250}$

$\rightarrow -\frac{t}{250} = \ln 2^{-2}$
 $\rightarrow -\frac{t}{250} = -2 \ln 2$
 $\rightarrow -\frac{t}{250} = -2 \cdot 0,693$
 $\rightarrow t = 1,386 \cdot 250$
 $t = 346,5$ (E)

(5) $C_F = C_0(1+i)^T$
 $43200 = 20000(1+0,08)^T$
 $\frac{432}{200} = (1+0,08)^T$
 $\log \frac{432}{200} = T$
 $1,08$

$\rightarrow T = \frac{\log 2,16}{\log 1,08}$
 $T = \frac{0,33}{0,03} = \frac{33}{3}$
 $T = 11$ (B)

432/200 = 2,16
 216/200 = 1,08
 108/200 = 0,54
 54/200 = 0,27
 27/200 = 0,135
 13,5/200 = 0,0675
 6,75/200 = 0,03375
 3,375/200 = 0,016875
 1,6875/200 = 0,0084375
 0,84375/200 = 0,00421875
 0,421875/200 = 0,002109375
 0,2109375/200 = 0,0010546875
 0,10546875/200 = 0,00052734375
 0,052734375/200 = 0,000263671875
 0,0263671875/200 = 0,0001318359375
 0,01318359375/200 = 0,00006591796875
 0,006591796875/200 = 0,000032958984375
 0,0032958984375/200 = 0,0000164794921875
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⑥ $F(t) = 100 \cdot (1,2)^t$

a) $F(0) = 100 \cdot 1,2^0$

$F(0) = 100$ reais

b) $2700 = 100(1,2)^t$

$27 = (1,2)^t$

$\log_{1,2} 27 = t$

$t = \frac{\log 3^3}{\log 12/10}$

$t = \frac{3 \log 3}{\log 3 + 2 \log 2 - \log 10}$

$t = \frac{3 \cdot 0,48}{0,48 + 0,6 - 1}$

$t = \frac{1,44}{0,08}$

$t = 18$ meses

⑦ $P(t) = 2000 \cdot 10^t$

~~460.000~~ $= 2000 \cdot 10^t$

$80 = 10^t$

$t = \log_{10} 80$

$\log_8 8 \cdot 10 = t$

$t = 3 \log 2 + \log 10$

$t = 3 \cdot 0,2 + 1$

(A)

⑧ $P(t) = 60(1 + e^{-0,05t})$

a) $P(0) = 60 \cdot (1 + e^0)$

$P(0) = 60 \cdot 2$

$P(0) = 120$ amperes

b) $60(1 + e^{-0,05t}) = 100$

$1 + e^{-0,05t} = 5/3$

$e^{-0,05t} = \frac{5}{3} - 1 = \frac{2}{3}$

$\ln e^{-0,05t} = \ln \frac{2}{3}$

$-0,05t = \ln 2 - \ln 3$

$-0,05t = 0,7 - 1,1$

$t = \frac{0,4}{0,05} = \frac{40}{5} = 8$ amperes

⑨ $n(t) = 800 \cdot 4^{0,02t}$

~~2400~~ $= 800 \cdot 4^{0,02t}$

$3 = 2^{0,04t}$

$\log 3 = 0,04t \log 2$

$0,48 = 0,04t \cdot 0,3$

$0,012t = 0,48$

$t = \frac{480}{12}$

$t = 40$ anos

$$P(0) = 120$$

70

$$P(t) = 120 \cdot e^{-0,005t}$$

$$80 = 120 \cdot e^{-0,005t}$$

$$\frac{2}{3} = e^{-0,005t}$$

$$\log_e \frac{2}{3} = -0,005t$$

$$-0,005t = \ln 2 - \ln 3$$

$$-0,005t = 0,7 - 1,1$$

$$-0,005t = -0,4$$

$$t = \frac{0,400}{0,005}$$

$$t = \frac{400}{5}$$

$$t = 80 \text{ dias}$$

m

11

$$N = \frac{20}{1 + 19 \cdot 10^{-0,5t}} = 10$$

$$2 = 1 + 19 \cdot 10^{-0,5t}$$

$$19 \cdot 10^{-0,5t} = 1$$

$$10^{-0,5t} = 19^{-1}$$

$$-0,5t = -1 \log 19$$

$$-0,5t = -1,28$$

$$t = \frac{1,28}{0,5} = \frac{256}{100}$$

$$t = 2,56$$

$$t = 3 \text{ semanas}$$

6ª parte

$$\textcircled{1} \text{ a) } \log_2 5 = \frac{\log 5}{\log 2} = \frac{\log 10 - \log 2}{\log 2} = \frac{1 - 0,3}{0,3} = \frac{0,7}{0,3} = \frac{7}{3}$$

$$\text{b) } \log_{12} 27 = \frac{\log 3^3}{\log 2^2 \cdot 3} = \frac{3 \log 3}{2 \log 2 + \log 3} = \frac{3 \cdot 0,48}{2 \cdot 0,3 + 0,48} = \frac{1,44}{1,08} = \frac{4}{3}$$

$$\textcircled{2} \text{ a) } \log_4 3 \cdot \log_5 4 \cdot \log_9 5 = \frac{\log 3}{\log 4} \cdot \frac{\log 4}{\log 5} \cdot \frac{\log 5}{2 \log 3} = \frac{1}{2}$$

$$\text{b) } 1$$

(3)

$$x = \log_3 2 \cdot \log_8 81$$

$$x = \frac{\log 2}{\log 3} \cdot \frac{4 \log 3}{3 \log 2} = \frac{4}{3}$$

(4)

$$\frac{1}{5} \log_5 10^4 = \frac{4}{5} \cdot \log_5 10 = \frac{4}{5} \cdot (\log_5 2 + \log_5 5) = \frac{4}{5} \cdot \left(\log_5 2 + 1 \right) = \frac{4}{5} \cdot \left(\log_5 2 + 1 \right)$$

$$= \frac{4}{5} (uv + 1) \quad \text{(B)}$$

(*)

$$\log_3 2 = u \Rightarrow \log 2 = u \log 3$$

$$\log_5 3 = v \Rightarrow \log 3 = v \log 5$$

$$\log 2 = u \cdot v \log 5$$

$$\log_5 2 = uv$$

(5)

$$5^n = 2 \Rightarrow \log_5 2 = n$$

$$\log_2 100 = 2 \log_2 10 = 2 (\log_2 2 + \log_2 5) = 2 + 2 \log_2 5 = 2 + \frac{2}{\log_5 2} = 2 + \frac{2}{n}$$

(6)

$$\log_{\sqrt{5}} 256 \cdot \log_{49} 125 \cdot \log_{\frac{1}{2}} 7$$

$$\log_{5^{1/2}} 2^8 \cdot \log_{7^2} 5^3 \cdot \log_{\frac{1}{2}} 7$$

$$\frac{8 \log 2}{\frac{1}{2}} \cdot \frac{3 \log 5}{2} \cdot \log_{\frac{1}{2}} 7$$

$$24 \cdot \log_5 2 \cdot \log_7 5 \cdot \frac{1}{\log_2 5}$$

$$7 \cdot 24 \cdot \log_5 2 \cdot \log_7 5 \cdot \frac{1}{\log_2 5}$$

$$-24 \cdot \log_5 2 \cdot \log_2 5$$

$$-24$$

(D)

$$\textcircled{7} \quad \log_{45} 2 = a$$

$$\log_{45} 3 = b$$

$$\textcircled{30)} \quad \log_3 2 = \frac{a}{b}$$

$$10) \quad \log_{90} 15 = \frac{\log_3 15}{\log_3 90} = \frac{\log_3 3 + \log_3 5}{\log_3 2 + 2 \log_3 3 + \log_3 5} = \textcircled{*}$$

$$20) \quad b = \log_{45} 3 = \frac{1}{\log_3 45} = \frac{1}{2 \log_3 3 + \log_3 5} = \frac{1}{2 + \log_3 5} = b$$

$$2 + \log_3 5 = \frac{1}{b} \Rightarrow \log_3 5 = \frac{1}{b} - 2$$

$$40) \quad \textcircled{*} \quad \log_{90} 15 = \frac{1 + \frac{1}{b} - 2}{\frac{a}{b} + 2 + \frac{1}{b} - 2} = \frac{\frac{1}{b} - 1}{\frac{a+1}{b}} = \frac{\frac{1-b}{b}}{\frac{a+1}{b}} = \frac{1-b}{a+1}$$