Soma dos Termos da P.G.

① a) $S_5 = \dfrac{81\left(\left(\frac{1}{3}\right)^5 - 1\right)}{\frac{1}{3} - 1} = \dfrac{81\left(\frac{1}{243} - \frac{1}{1}\right)}{\frac{1}{3} - \frac{1}{1}} = \dfrac{81 \cdot \frac{(1-243)}{243}}{\frac{1-3}{3}}$

$= \dfrac{\cancel{81} \cdot \frac{(-242)}{\cancel{243}\,3}}{-\frac{2}{3}} = -\dfrac{\overset{121}{\cancel{242}}}{\cancel{3}} \cdot \left(-\dfrac{\cancel{3}}{\cancel{2}}\right) = 121$

b) $S_5 = \dfrac{\frac{1}{1000}\left(10^5 - 1\right)}{10 - 1} = \dfrac{\frac{1}{1000} \cdot \overset{11\,111}{\cancel{99999}}}{\cancel{9}} = \dfrac{11.111}{1000} = 11,111$

② a) $64 = (-2)(-2)^{n-1}$

$64 = (-2)^n$

$2^6 = (-2)^n$

$n = 6$

$S_6 = \dfrac{-2\left((-2)^6 - 1\right)}{-2 - 1} = \dfrac{-2(64-1)}{-3}$

$S_6 = -\dfrac{2 \cdot 63}{-3} = 42$

b) $768 = 3 \cdot 4^{n-1}$

$\dfrac{768}{3} = 4^{n-1}$

$256 = 4^{n-1}$

$\cancel{4}^4 = \cancel{4}^{n-1}$

$n = 5$

$S_5 = \dfrac{3\left(4^5 - 1\right)}{4 - 1} = \dfrac{\cancel{3} \cdot (1024 - 1)}{\cancel{3}}$

$S_5 = 1023$

③ $S_n = \dfrac{2\left(3^n - 1\right)}{3 - 1} = \dfrac{\cancel{2}\left(3^n - 1\right)}{\cancel{2}} = 3^n - 1 = 19\,682$

$\Rightarrow \cancel{3}^n = 19\,683 = \cancel{3}^9$

$\therefore n = 9$

(4) $(a_1, 2a_1, 4a_1, 8a_1, 16a_1)$

$$S_5 = \frac{a_1(2^5 - 1)}{2 - 1} = 31\,a_1 = 74400$$

$$a_1 = 2400$$

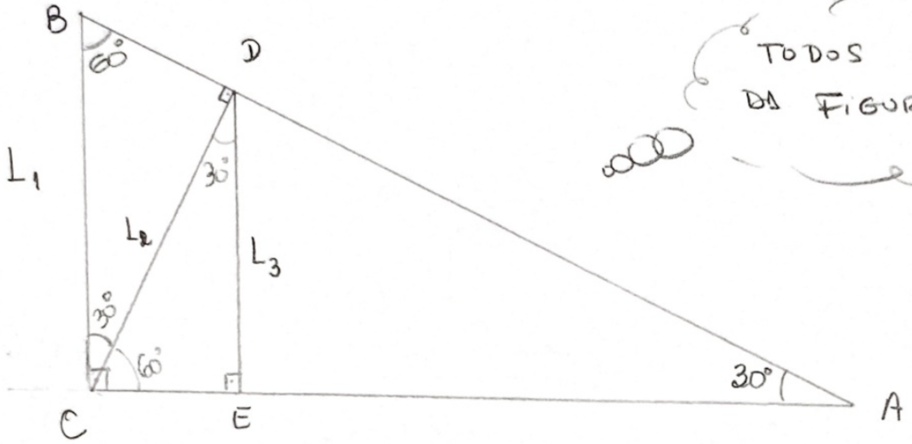
(5) $a_1 = 1$

$q = 2$

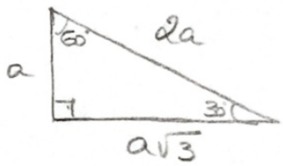
$n = 1001$

$$S_{1001} = \frac{1(2^{1001} - 1)}{2 - 1} = 2^{1001} - 1 \quad \text{(A)}$$

(6)



TODOS OS TRÂNGULOS DA FIGURA SÃO EGÍPCIOS



No ΔBCD: $L_1 \rightarrow$ HIPOTENUSA; $L_2 \rightarrow$ ADJACENTE A 30° $\Rightarrow L_2 = \frac{L_1}{2}\sqrt{3}$

P.G. DE RAZÃO $q = \frac{\sqrt{3}}{2}$

No ΔCDE: $L_2 \rightarrow$ HIPOTENUSA; $L_3 \rightarrow$ ADJACENTE DE 30° $\Rightarrow L_3 = \frac{L_2}{2}\sqrt{3}$

$$(L_1, L_2, L_3, L_4, \ldots) = \left(L_1, L_1 \cdot \frac{\sqrt{3}}{2}, L_1 \cdot \left(\frac{\sqrt{3}}{2}\right)^2, \ldots\right)$$

$$L_9 = L_1 \cdot \left(\frac{\sqrt{3}}{2}\right)^8 = L_1 \cdot \frac{81}{256} \Rightarrow \boxed{\frac{L_9}{L_1} = \frac{81}{256}} \quad \text{(C)}$$

OBS: Como $\frac{L_9}{L_1}$ é uma razão, poderíamos ter chutado um valor para L_1.

7) a) $S_4 = \dfrac{x\left(\left(\frac{1}{3}\right)^4 - 1\right)}{\frac{1}{3} - 1} = \dfrac{x\left(\frac{1}{81} - 1\right)}{\frac{1}{3} - 1} = \dfrac{x \cdot \left(-\frac{80}{81}\right)}{-\frac{2}{3}}$ PG 3

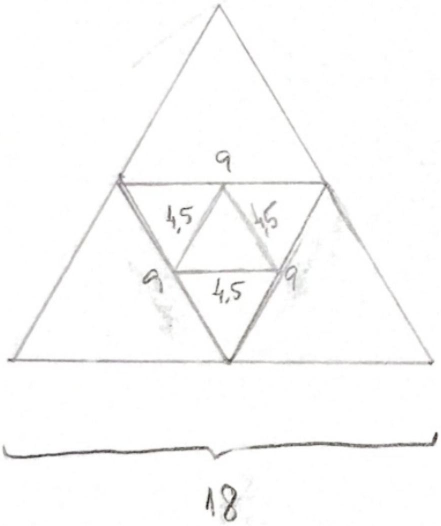
$= x \cdot \left(-\dfrac{\cancel{80}^{\,40}}{\cancel{81}_{\,27}}\right) \cdot \left(-\dfrac{\cancel{3}}{\cancel{2}}\right) = \dfrac{40x}{\cancel{27}} = \dfrac{200}{\cancel{27}} \quad \therefore \quad x = 5$

b) $S_8 = \dfrac{10x\left(2^8 - 1\right)}{2 - 1} = 10x \cdot 255 = 7650$

$x = 3$

2ª Parte

1)



$P_1 = 18 + 18 + 18 = 54 \quad \div 2$

$P_2 = 9 + 9 + 9 = 27 \quad \div 2$

$P_3 = 4,5 + 4,5 + 4,5 = 13,5$

$\vdots$

PG infinita de razão $q = \dfrac{1}{2}$

$S_\infty = \dfrac{54}{1 - \frac{1}{2}} = \dfrac{54}{\frac{1}{2}} = 54 \cdot \dfrac{2}{1}$

$S_\infty = 108$

2) $\dfrac{3x}{20 + 5x} = \dfrac{2x - 5}{3x} \Rightarrow 9x^2 = 40x - 100 + 10x^2 - 25x$

$\Rightarrow x^2 + 15x - 100 = 0$

$S = -15 \qquad x_1 = -20$

$P = -100 \qquad x_2 = 5$

Como os termos são negativos: $x = -20$

$(-80, -60, -45, \ldots)$

$q = \dfrac{-60}{-80} = \dfrac{3}{4} < 1$

$S_\infty = \dfrac{-80}{1 - 3/4} = \dfrac{-80}{1/4} = -80 \cdot \dfrac{4}{1} = -320$

③ a) $S_\infty = \dfrac{5x}{1-\frac{2}{3}} = \dfrac{5x}{\frac{1}{3}} = 5x \cdot \dfrac{3}{1} = 15x = 20$

$x = \dfrac{20}{15} = \dfrac{4}{3}$

b) $S_\infty = \dfrac{x^2}{1-\left(-\frac{1}{2}\right)} = \dfrac{x^2}{1+\frac{1}{2}} = \dfrac{x^2}{3/2} = \dfrac{2x^2}{3} = 6 \Rightarrow x^2 = 9$

$\therefore x = \pm 9$

4) a) $S_\infty = \dfrac{\frac{1}{3}}{1-\frac{2}{3}} = \dfrac{\frac{1}{3}}{\frac{1}{3}} = 1$

b) $S_\infty = \underbrace{\left(\frac{1}{3} + \frac{1}{9} + \cdots\right)}_{\frac{\frac{1}{3}}{1-\frac{1}{3}}} + \underbrace{\left(\frac{1}{5} + \frac{1}{25} + \cdots\right)}_{\frac{\frac{1}{5}}{1-\frac{1}{5}}} = \dfrac{\frac{1}{3}}{\frac{2}{3}} + \dfrac{\frac{1}{5}}{\frac{4}{5}}$

$= \cancel{\frac{1}{3}} \cdot \frac{\cancel{3}}{2} + \cancel{\frac{1}{5}} \cdot \frac{\cancel{5}}{4} = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

⑤ $S_\infty - a_1 = a_1 \Rightarrow S_\infty = 2a_1 \Rightarrow \dfrac{\cancel{a_1}}{1-q} = 2\cancel{a_1}$

$\Rightarrow 1 - q = \dfrac{1}{2} \Rightarrow 1 - \dfrac{1}{2} = q \Rightarrow q = \dfrac{1}{2}$

$a_1 + a_1 \cdot \dfrac{1}{2} = 9 \Rightarrow \dfrac{3a_1}{2} = 9 \Rightarrow \boxed{a_1 = 6}$

$a_3 = 6 \cdot \left(\dfrac{1}{2}\right)^2 = \dfrac{6}{4} = \dfrac{3}{2}$

(6) $\underbrace{1+\frac{1}{2}+\frac{1}{4}+\dots+\left(\frac{1}{2}\right)^{10}}_{\text{Finita!}}$

$$S_{10} = \frac{1\left(\left(\frac{1}{2}\right)^{10}-1\right)}{\frac{1}{2}-1} = \frac{\frac{1}{2^{10}}-1}{-\frac{1}{2}} \begin{matrix}\cdot(-1)\\ \cdot(-1)\end{matrix}$$

$$S_{10} = \frac{1-\frac{1}{2^{10}}}{\frac{1}{2}} = \left(1-\frac{1}{2^{10}}\right)\cdot\frac{2}{1} \quad \text{(C)}$$

PG 5

(7) $\left(x, \frac{x}{2}, \frac{x}{4}, \frac{x}{8}, \dots\right)$

$$x+\frac{x}{4}+\frac{x}{16}+\dots-\left(\frac{x}{2}+\frac{x}{8}+\frac{x}{32}+\dots\right)=\frac{32}{3}$$

$$\frac{x}{1-\frac{1}{4}}-\frac{\frac{x}{2}}{1-\frac{1}{4}}=\frac{x}{\frac{3}{4}}-\frac{\frac{x}{2}}{\frac{3}{4}}=\frac{4x}{3}-\frac{\cancel{4}^{2}x}{\cancel{6}_{3}}$$

$$=\frac{2x}{\cancel{3}}=\frac{32}{\cancel{3}} \quad \therefore \quad x=16 \quad \text{(C)}$$

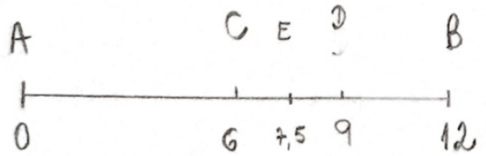
(8) $(20, 10, 5, \dots)$

$$S_\infty = \frac{20}{1-\frac{1}{2}} = \frac{20}{1/2} = 40\text{ kg}$$

No máximo, ela perderá 40 kg, e ficará com 80 kg

(D)

(9)

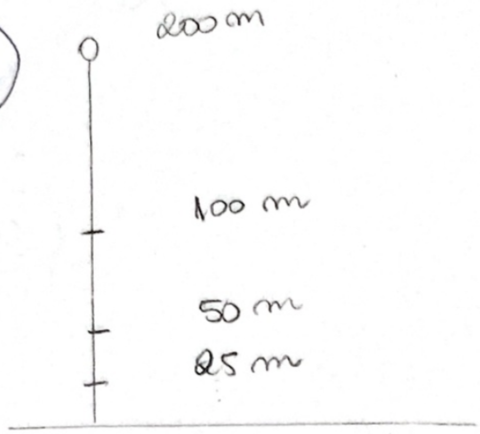


$$\left(12,\ \overset{=6}{12-6},\ \overset{=9}{6+3},\ \overset{=7,5}{9-1,5},\ \dots\right)$$

A sequência será: $12-\frac{12}{2}+\frac{12}{4}-\frac{12}{8}+\dots$

$$S_\infty = \frac{12}{1-\left(-\frac{1}{2}\right)} = \frac{12}{1+\frac{1}{2}} = \frac{12}{\frac{3}{2}} = \cancel{12}^{4}\cdot\frac{2}{\cancel{3}} = 8 \quad \text{(D)}$$

10



PG 6

$$S_\infty = 200 + 2(100 + 50 + 25 + \dots)$$

$$S_\infty = 200 + 2 \cdot \frac{100}{1 - \frac{1}{2}}$$

$$S_\infty = 200 + 2 \cdot \frac{100}{\frac{1}{2}} = 200 + 200 \cdot \frac{2}{1}$$

$$\therefore S_\infty = 600\ m$$