Gabarito da revisão

PFV (PG e Esfera)

① $a_2 = 32$

$a_7 = 243$

$a_7 = a_2 \cdot q^5$

$243 = 32 \cdot q^5$

$q^5 = \left(\frac{3}{2}\right)^5$

$q = \frac{3}{2}$

$a_4 = a_2 \cdot q^2$

$a_4 = 32 \cdot \left(\frac{3}{2}\right)^2$

$a_4 = \frac{8}{\cancel{32}} \cdot \frac{9}{\cancel{4}}$

$a_4 = 72$

② $PG : (9, 3, 1, \dots)$

$a_8 = \frac{1}{x}$ e $q = \frac{1}{3}$

$a_8 = a_3 \cdot q^5$

$\frac{1}{x} = 1 \cdot \left(\frac{1}{3}\right)^5$

$\frac{1}{x} = \frac{1}{243} \Rightarrow \boxed{x = 243}$

③ $(5, 8, 14) \Rightarrow (5-x, 8-x, 14-x)$ PG

$\frac{8-x}{5-x} \neq \frac{14-x}{8-x}$

$64 - 16x + x^2 = 70 - 5x - 14x + x^2$

$3x = 6$

$x = 2$

PG: $(5-2, 8-2, 14-2, \ldots)$

↳ $(3, 6, 12, \ldots) \rightarrow q = \frac{6}{3} = 2$

$$S_{10} = \frac{a_1 \cdot (q^n - 1)}{q - 1}$$

$$S_{10} = \frac{3 \cdot (2^{10} - 1)}{2 - 1}$$

$$S_{10} = 3 \cdot (1024 - 1)$$

$$S_{10} = 3 \cdot 1023$$

$$\boxed{S_{10} = 3069}$$

(4) PG

$n = 3$

$q = 4$

$S_3 = 315$

$(x, 4x, 16x)$

$x + 4x + 16x = 315$

$21x = 315$

$\boxed{x = 15}$

(5) PG

$n = 6$

$a_3 = 180$

$a_5 = 1620$

a) $a_5 = a_3 \cdot q^2$

$1620 = 180 \cdot q^2$

$q^2 = 9$

$q = 3$

$a_3 = a_1 \cdot q^2$

$180 = a_1 \cdot 9$

$\boxed{a_1 = 20}$

b) $S_6 = \dfrac{a_1 \cdot (q^6 - 1)}{q - 1}$

$S_6 = \dfrac{20 \cdot (3^6 - 1)}{3 - 1}$

$S_6 = \dfrac{20\ (729 - 1)}{2}$

$\boxed{S_6 = 7280}$

(6) $S = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$

↳ Soma da PG infinita

$$q = \frac{1}{2}$$

$$S = \frac{a_1}{1-q} = \frac{1}{1-\frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2$$

$$\log_2 S = \log_2 2 = 1$$

(7) $V_1 = a^3$

$$V_2 = \left(\frac{a}{2}\right)^3 = \frac{a^3}{8}$$

$$V_3 = \left(\frac{a}{4}\right)^3 = \frac{a^3}{64}$$

⋮

$$V = V_1 + V_2 + V_3 + \dots$$

↳ Soma da PG infinita

$$q = \frac{1}{8}$$

$$V = \frac{a_1}{1-q} = \frac{a^3}{1-\frac{1}{8}} = \frac{a^3}{\frac{7}{8}}$$

$$\boxed{V = \frac{8a^3}{7}}$$

(8)

PG	PA
$q = 2$	$r = ?$
$a_1 = 1$	$a_1 = 1$
$S_4 = x$	$S_4 = x$
$n = 4$	$n = 4$

$$S_4 = \frac{a_1(q^4 - 1)}{q - 1}$$

$$S_4 = \frac{1 \cdot (2^4 - 1)}{2 - 1}$$

$$S_4 = 15$$

$$S_4 = (a_1 + a_4) \cdot \frac{4}{2}$$

$$15 = (1 + a_4) \cdot 2$$

$$15 = 2 + 2a_4$$

$$2a_4 = 13$$

$$a_4 = \frac{13}{2}$$

PA: $a_4 = a_1 + 3r$

$$\frac{13}{2} = 1 + 3r$$

$$13 = 2 + 6r$$

$$6r = 11$$

$$\boxed{r = \frac{11}{6}}$$

(9) 1) Água doce superficial: $R_1 = \frac{58}{2} = 29$

2) Água doce: $R_2 = \frac{406}{2} = 203$

$$\frac{V_1}{V_2} = \frac{R_1^3}{R_2^3} = \frac{29^3}{203^3} = \left(\frac{1}{7}\right)^3 = \boxed{\frac{1}{343}}$$

(10) Cilindro:

$r = 5$

$h = 20$

$h_{água} = 19$

Esfera

$R = 1$

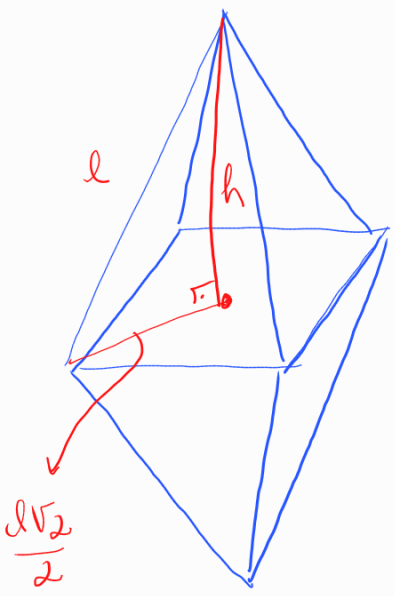
$V = \frac{4}{3}\pi \cdot 1^3 = 4$

$V_{sem\ água} = \pi \cdot 5^2 \cdot (20-19) = 25 \cdot 3 = 75$

$75 \div 4 = 18{,}75$

R: 18

(11) $V_{octaedro} = 9\sqrt{2}$



$$h^2 + \left(\frac{\ell\sqrt{2}}{2}\right)^2 = \ell^2$$

$$h^2 + \frac{\ell^2}{2} = \ell^2$$

$$h^2 = \frac{\ell^2}{2}$$

$$h = \frac{\ell}{\sqrt{2}}$$

$V =$ raio da esfera

$$V_E = \frac{4}{3}\pi \cdot R^3 = \frac{4}{3} \cdot 3 \cdot R^3 = 4R^3$$

$$V_E = 4 \cdot \left(\frac{3}{\sqrt{2}}\right)^3$$

$$V_{OC} = 2 \cdot \frac{1}{3} \cdot \ell^2 \cdot h$$

$$V_E = 4 \cdot \frac{27}{2\sqrt{2}} \cdot \sqrt{2}$$

$$9\sqrt{2} = \frac{2}{3} \cdot \ell^2 \cdot \frac{\ell}{\sqrt{2}}$$

$$V_E = 27\sqrt{2}$$

$$\ell^3 = 27$$

$$\ell = 3 \Rightarrow h = \frac{3}{\sqrt{2}}$$

$$V_E - V_{OC} = 27\sqrt{2} - 9\sqrt{2} = \boxed{18\sqrt{2}\ cm^3}$$

(12) $R_E = 6$

$H_{CIL} = 8$

$V_E = V_{CIL}$

$\frac{4}{3}\pi R_E^3 = \pi R^2 \cdot H_{CIL}$

$\frac{4}{3} \cdot \pi \cdot 6^3 = \pi R^2 \cdot 8$

$R^2 = \frac{6^3}{6} = 6^2$

$\boxed{R = 6 \text{ cm}}$

(13) $R = 3$

$\rightarrow$ aresta $= 2R$

$V_{AR} = V_{CUBO} - V_E$

$V_{AR} = (2R)^3 - \frac{4}{3}\pi R^3$

$V_{AR} = 6^3 - \frac{4}{3}\pi \cdot 3^3 = \boxed{216 - 36\pi \text{ cm}^3}$

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$$A = A_{L(CIL)} + A_E$$

$$A = 2\pi RH + 4\pi R^2$$

$$A = 2 . 3 . 0,45 . 1 + 4 . 3 . (0,45)^2$$

$$A = 2,7 + 2,43$$

$$\boxed{A = 5,13\ m^2}$$

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Cone : $r = 10$

$h = 30$

$$V = \frac{1}{3}\pi . 10^2 . 30$$

$$V = 1000 . \pi$$

$$V = 3100\ cm^3$$

Esferas: $r = \frac{1,5}{2} = \frac{3}{4}$

$$V = \frac{4}{3}\pi \cdot \left(\frac{3}{4}\right)^3$$

$$V = \frac{4}{3} \cdot 3,1 \cdot \frac{27}{64}\ 16\ 9$$

$$V \approx 1,74 \times 200$$

$$V_{200} = 348 \ cm^3$$

$$V_{LÍQ} = V_{CONE} - V_{200}$$

$$V_{LÍQ} = 3100 - 348 = \boxed{2752\ cm^3}$$