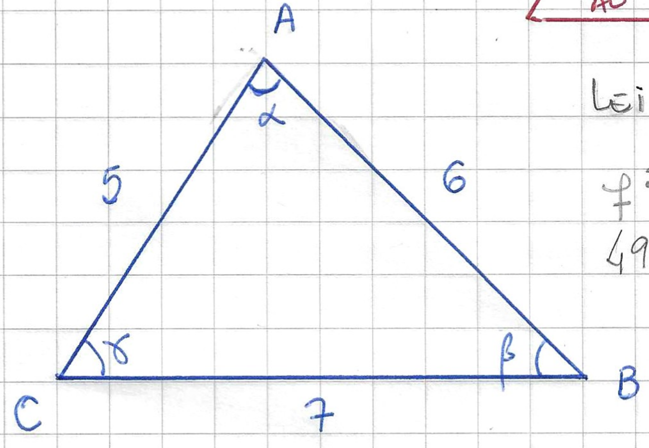
Matemática II - lista 3

O MAIOR ÂNGULO É SEMPRE OPOSTO AO MAIOR LADO.

①



LEI DOS COSSENOS:

$7^2 = 5^2 + 6^2 - 2 \cdot 5 \cdot 6 \cdot \cos\alpha$

$49 = 25 + 36 - 60\cos\alpha$

$49 - 25 - 36 = -60 \cdot \cos\alpha$

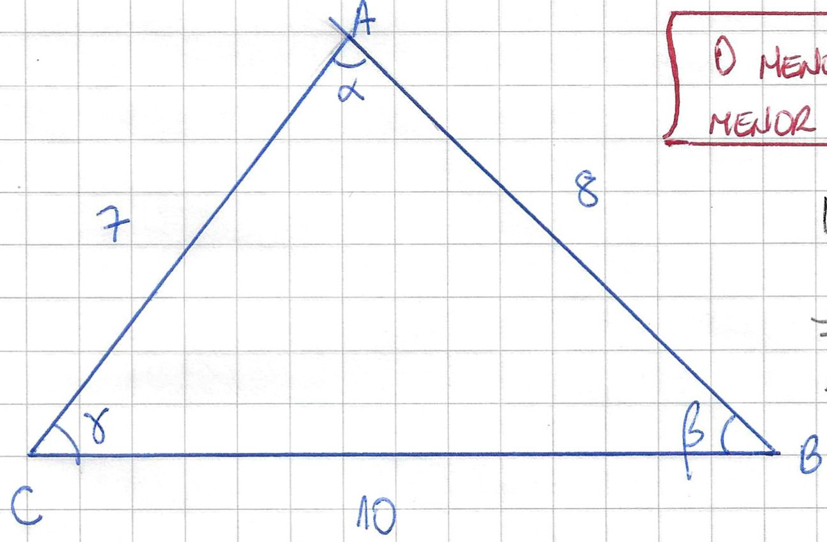
$-12 = -60\cos\alpha \quad \cdot(-1)$

$12 = 60\cos\alpha \Rightarrow \cos\alpha = \dfrac{12 \div 12}{60 \div 12}$

$\therefore \boxed{\cos\alpha = \dfrac{1}{5}}$

O MENOR ÂNGULO É OPOSTO AO MENOR LADO.

②



LEI DOS COSSENOS:

$7^2 = 8^2 + 10^2 - 2 \cdot 8 \cdot 10 \cdot \cos\beta$

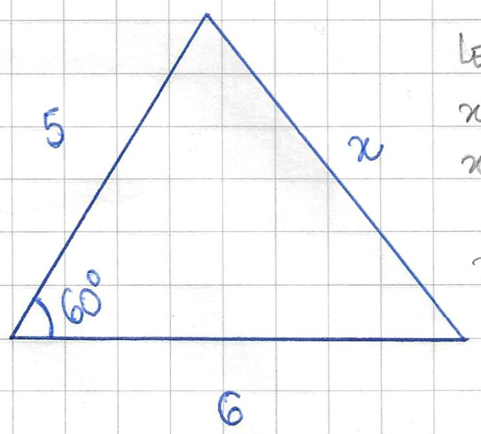
$49 = 64 + 100 - 160 \cdot \cos\beta$

$49 - 64 - 100 = -160\cos\beta$

$-115 = -160\cos\beta \cdot (-1)$

$\Rightarrow 115 = 160\cos\beta \Rightarrow \cos\beta = \dfrac{115 \div 5}{160 \div 5}$ $\quad \therefore \boxed{\cos\beta = \dfrac{23}{32}}$

③



LEI DOS COSSENOS:

$x^2 = 5^2 + 6^2 - 2 \cdot 5 \cdot 6 \cdot \cos 60°$

$x^2 = 25 + 36 - 60 \cdot \dfrac{1}{2}$

$x^2 = 25 + 36 - 30$

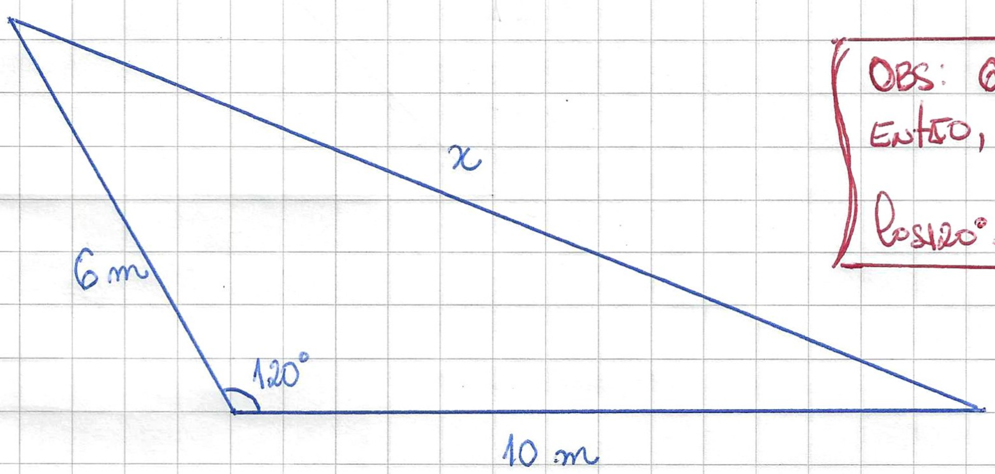
$x^2 = 31$

$\therefore \boxed{x = \sqrt{31}}$

4) PARA QUE O TRIÂNGULO EXISTA, É NECESSÁRIO QUE SEJA RESPEITADA A DESIGUALDADE TRIANGULAR.

$a+b>c$, $b+c>a$ e $a+c>b$.

COMO $2+3=5$, NÃO EXISTE TAL TRIÂNGULO.

5) 

OBS: $60° + 120° = 180°$
ENTÃO,
$\cos 120° = -\cos 60°$

LEI DOS COSSENOS:

$$x^2 = 6^2 + 10^2 - 2 \cdot 6 \cdot 10 \cdot \cos 120°$$
$$x^2 = 36 + 100 - 120 \cdot \left(-\frac{1}{2}\right)$$
$$x^2 = 36 + 100 + 60$$
$$x^2 = 196 \Rightarrow x = \sqrt{196} \quad \therefore \boxed{x = 14\ m}$$

6) NUM TRIÂNGULO OBTUSÂNGULO, TEMOS: $a^2 > b^2 + c^2$, ONDE a É A MEDIDA DO MAIOR LADO

SE x FOR O MAIOR LADO, ENTÃO: $x^2 > 3^2 + 4^2 \Rightarrow x^2 > 25 \Rightarrow x > 5$
ALÉM DISSO, A DESIG. TRIANG. GARANTE QUE: $3+4>x \Rightarrow x<7$

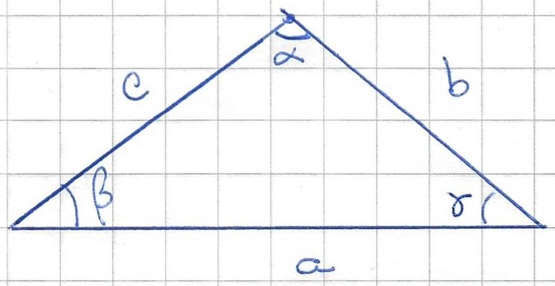
LOGO, NESTE CASO, $5<x<7$.

SE 4 FOR O MAIOR LADO: $4^2 > x^2 + 3^2 \Rightarrow 16 > x^2 + 9 \Rightarrow 16-9 > x^2$
$\Rightarrow 7 > x^2 \Rightarrow x < \sqrt{7}$.
PELA DESIG. TRIANG.: $x+3>4 \Rightarrow x > 4-3 \Rightarrow x>1$

NESTE CASO, $1<x<\sqrt{7}$

C

7



Lei dos senos:

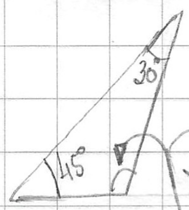
$$\frac{a}{\operatorname{sen}\alpha} = \frac{b}{\operatorname{sen}\beta} = \frac{c}{\operatorname{sen}\gamma}$$

a) $$\frac{4}{\operatorname{sen}30^\circ} = \frac{8}{\operatorname{sen}\beta} \iff 4\operatorname{sen}\beta = 8\operatorname{sen}30^\circ \iff 4\operatorname{sen}\beta = \overset{4}{\cancel{8}} \cdot \frac{1}{\cancel{2}_{1}}$$

$$\iff 4\operatorname{sen}\beta = 4 \Rightarrow \operatorname{sen}\beta = \frac{4}{4} \therefore \boxed{\operatorname{sen}\beta = 1}$$

b) $$\frac{\sqrt{2}}{\operatorname{sen}\alpha} = \frac{2}{\operatorname{sen}45^\circ} \iff 2\operatorname{sen}\alpha = \sqrt{2}\cdot\operatorname{sen}45^\circ \iff 2\operatorname{sen}\alpha = \sqrt{2}\cdot\frac{\sqrt{2}}{2}$$

$$\iff 2\operatorname{sen}\alpha = \frac{\cancel{2}}{\cancel{2}} \iff \operatorname{sen}\alpha = \frac{1}{2} \Rightarrow \alpha = 30^\circ$$



$$\boxed{\gamma = 105^\circ}$$

c) $$\frac{\sqrt{3}}{\operatorname{sen}\alpha} = \frac{10}{\operatorname{sen}\gamma} \iff 10\operatorname{sen}\alpha = \sqrt{3}\operatorname{sen}\gamma \iff 10\operatorname{sen}\alpha = \sqrt{3}\cdot\frac{\sqrt{3}}{3}$$

$$\iff 10\operatorname{sen}\alpha = \frac{\cancel{3}}{\cancel{3}} \Rightarrow \operatorname{sen}\alpha = \frac{1}{10}$$

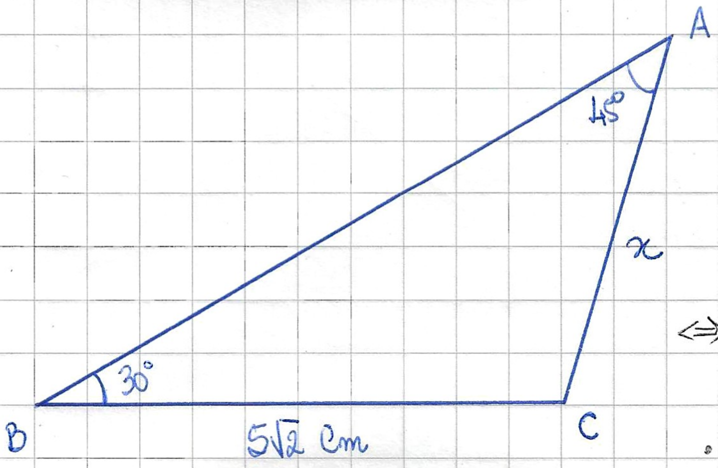
Relação Fundamental: $\operatorname{sen}^2\alpha + \cos^2\alpha = 1$

$$\Rightarrow \left(\frac{1}{10}\right)^2 + \cos^2\alpha = 1 \Rightarrow \frac{1}{100} + \cos^2\alpha = 1 \Rightarrow \cos^2\alpha = 1 - \frac{1}{100}$$

$$\Rightarrow \cos^2\alpha = \frac{99}{100} \Rightarrow \cos\alpha = \sqrt{\frac{99}{100}} \Rightarrow \cos\alpha = \frac{\sqrt{9\cdot 11}}{\sqrt{100}}$$

$$\therefore \boxed{\cos\alpha = \frac{3\sqrt{11}}{10}}$$

(8)



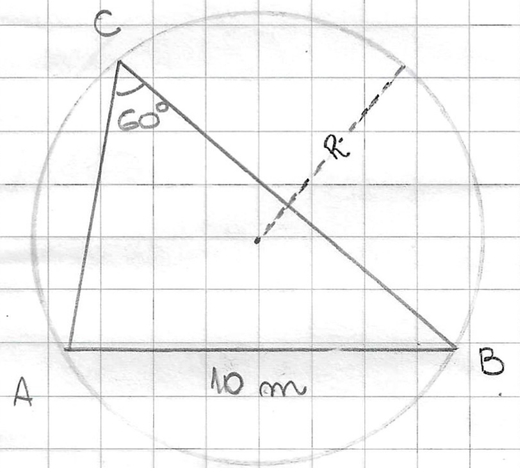
Lei dos senos:

$$\frac{x}{\text{sen}30°} = \frac{5\sqrt{2}}{\text{sen}45°} \Leftrightarrow x\,\text{sen}45° = 5\sqrt{2}\,\text{sen}30°$$

$$\Leftrightarrow x \cdot \frac{\sqrt{2}}{2} = 5\sqrt{2} \cdot \frac{1}{2} \Leftrightarrow x = \frac{5\sqrt{2}}{\cancel{2}} \cdot \frac{\cancel{2}}{\sqrt{2}}$$

$\therefore \boxed{x = 5\text{ cm}}$

(9)



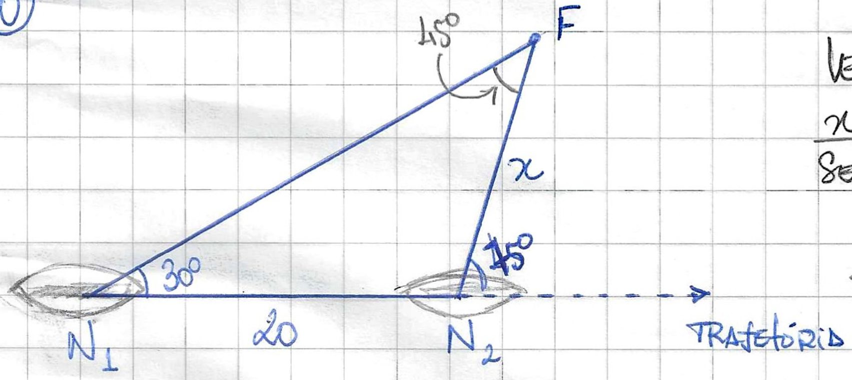
Lei dos senos:

$$\frac{10}{\text{sen}60°} = 2R \Leftrightarrow \frac{10}{\frac{\sqrt{3}}{2}} = 2R$$

$$10 \cdot \frac{2}{\sqrt{3}} = 2R \Rightarrow R = \frac{\overset{10}{\cancel{20}}}{\cancel{2}\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\boxed{R = \frac{10\sqrt{3}}{3}\text{ m}}$$

(10)



Lei dos senos:

$$\frac{x}{\text{sen}30°} = \frac{20}{\text{sen}45°}$$

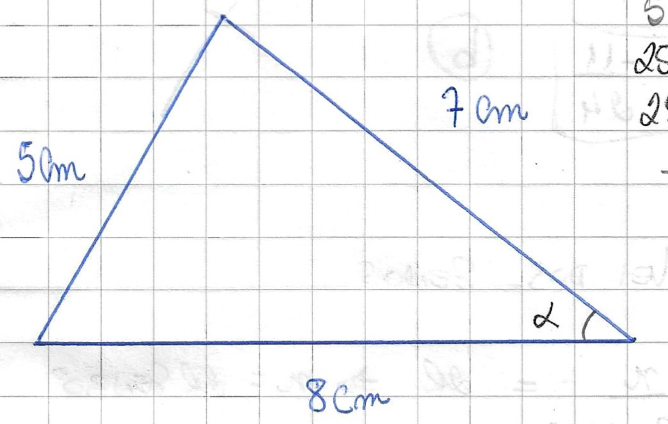
$$\Leftrightarrow x\,\text{sen}45° = 20\,\text{sen}30°$$

$$x\frac{\sqrt{2}}{2} = 20 \cdot \frac{1}{2}$$

$$\Leftrightarrow \frac{x\sqrt{2}}{2} = 10 \Leftrightarrow x = \frac{10 \cdot 2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \Rightarrow x = \frac{\overset{10}{\cancel{20}}\sqrt{2}}{\cancel{2}}$$

$\therefore \boxed{x = 10\sqrt{2}\text{ milhas}}$

11



LEI DOS COSSENOS:

$5^2 = 7^2 + 8^2 - 2 \cdot 7 \cdot 8 \cdot \cos\alpha$

$25 = 49 + 64 - 112 \cdot \cos\alpha$

$25 - 49 - 64 = -112 \cos\alpha$

$-88 = -112 \cos\alpha \quad \cdot(-1)$

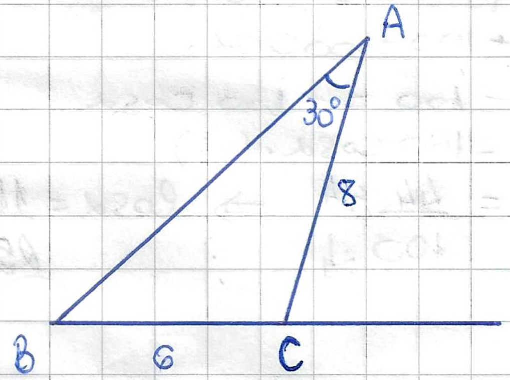
$\cos\alpha = \dfrac{88^{\div 8}}{112_{\div 8}} \Rightarrow \boxed{\cos\alpha = \dfrac{11}{14}}$

RELAÇÃO FUNDAMENTAL: $\operatorname{sen}^2\alpha + \cos^2\alpha = 1$

$\operatorname{sen}^2\alpha + \left(\dfrac{11}{14}\right)^2 = 1 \Rightarrow \operatorname{sen}^2\alpha = 1 - \dfrac{121}{196} \Rightarrow \operatorname{sen}^2\alpha = \dfrac{75}{196}$

$\Rightarrow \operatorname{sen}\alpha = \sqrt{\dfrac{75}{196}} \Rightarrow \operatorname{sen}\alpha = \dfrac{\sqrt{25 \cdot 3}}{14} \therefore \boxed{\operatorname{sen}\alpha = \dfrac{3\sqrt{5}}{14}}$

12



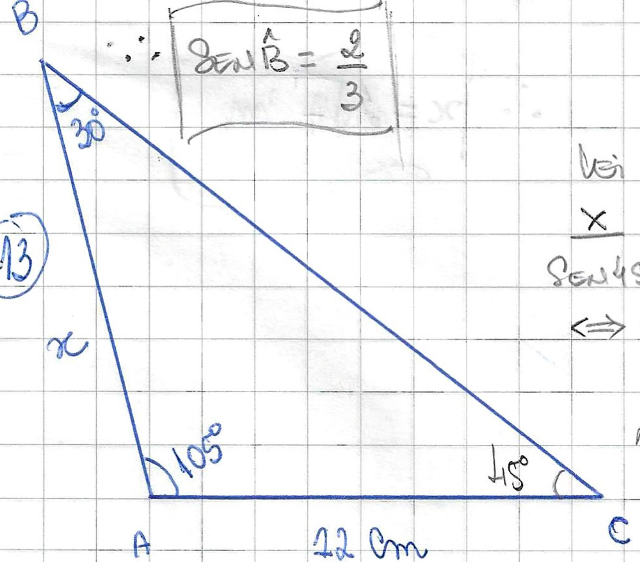
LEI DOS SENOS:

$\dfrac{8}{\operatorname{sen}\hat{B}} = \dfrac{6}{\operatorname{sen}30°} \Leftrightarrow 6 \operatorname{sen}\hat{B} = 8 \operatorname{sen}30°$

$\Leftrightarrow 6 \cdot \operatorname{sen}\hat{B} = \cancel{8}^{4} \cdot \dfrac{1}{\cancel{2}} \Leftrightarrow \operatorname{sen}\hat{B} = \dfrac{4}{6}{}^{\div 2}_{\div 2}$

$\therefore \boxed{\operatorname{sen}\hat{B} = \dfrac{2}{3}}$

13



LEI DOS SENOS:

$\dfrac{x}{\operatorname{sen}45°} = \dfrac{12}{\operatorname{sen}30°} \Leftrightarrow x \operatorname{sen}30° = 12 \operatorname{sen}45°$

$\Leftrightarrow x \cdot \dfrac{1}{\cancel{2}} = 12 \dfrac{\sqrt{2}}{\cancel{2}} \Rightarrow x \simeq 12 \cdot 1{,}4 = 16{,}8 \text{ Cm}$

NA ESCALA 1:10 000, 16,8 Cm CORRESPONDE A 1,68 Km

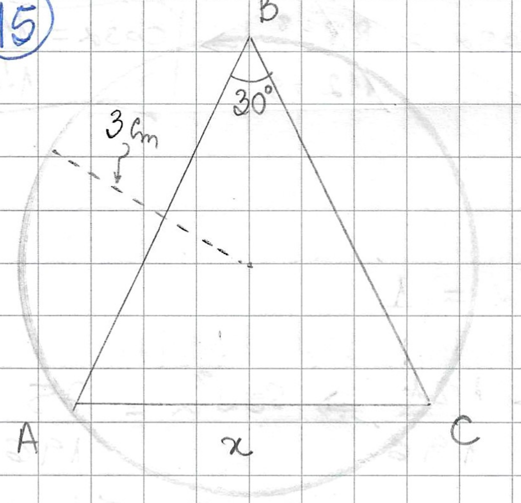
e

(14) $6^2 = 3^2 + 4^2 - 2 \cdot 3 \cdot 4 \cos\alpha$

$36 - 9 - 16 = -24 \cos\alpha$

$11 = -24 \cos\alpha \Rightarrow \boxed{\cos\alpha = \frac{-11}{24}}$ (b)

(15)

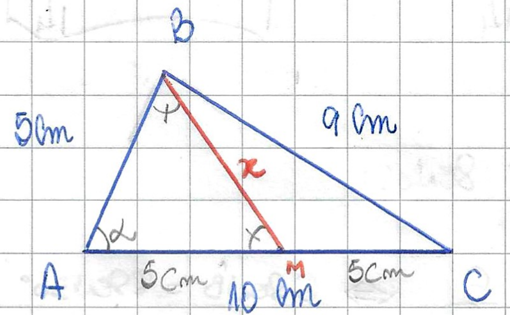


LEI DOS SENOS

$\frac{x}{\text{sen}30°} = 2R \Rightarrow x = 2R\,\text{sen}30°$

$x = \cancel{2} \cdot 3 \cdot \frac{1}{\cancel{2}} \therefore \boxed{x = 3\text{ cm}}$

(16)



LEI DOS COSSENOS:

$9^2 = 5^2 + 10^2 - 2 \cdot 5 \cdot 10 \cdot \cos\alpha$

$81 = 25 + 100 - 100\cos\alpha$

$81 - 25 - 100 = -100\cos\alpha$

$-44 = -100\cos\alpha \cdot (-1)$

$\cos\alpha = \frac{44 : 4}{100 : 4} \Rightarrow \cos\alpha = \frac{11}{25}$

$x^2 = 5^2 + 5^2 - 2 \cdot 5 \cdot 5 \cos\alpha$

$x^2 = 25 + 25 - \cancel{50}^{2} \cdot \frac{11}{\cancel{25}}$

$x^2 = 25 + 25 - 2 \cdot 11$

$x^2 = 28 \Rightarrow x = \sqrt{28} = \sqrt{4 \cdot 7} \therefore \boxed{x = 2\sqrt{7}\text{ cm}}$

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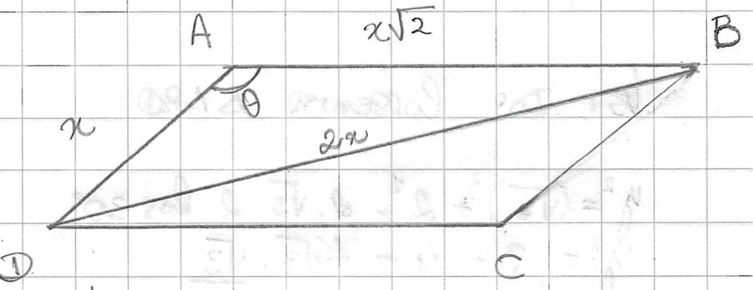


LEI DOS COSSENOS ΔDAB.

$$(2x)^2 = x^2 + (x\sqrt{2})^2 - 2x \cdot x\sqrt{2} \cos\theta$$
$$4x^2 = x^2 + 2x^2 - 2\sqrt{2}\, x^2 \cos\theta$$
$$4x^2 - 3x^2 = -2\sqrt{2}\, x^2 \cos\theta$$
$$x^2 = -2\sqrt{2}\, x^2 \cos\theta \quad \cdot(-1)$$
$$\Rightarrow \cos\theta = \frac{\cancel{x^2}}{2\sqrt{2}\,\cancel{x^2}} = \frac{1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \Rightarrow \boxed{\cos\theta = \frac{\sqrt{2}}{4}}$$

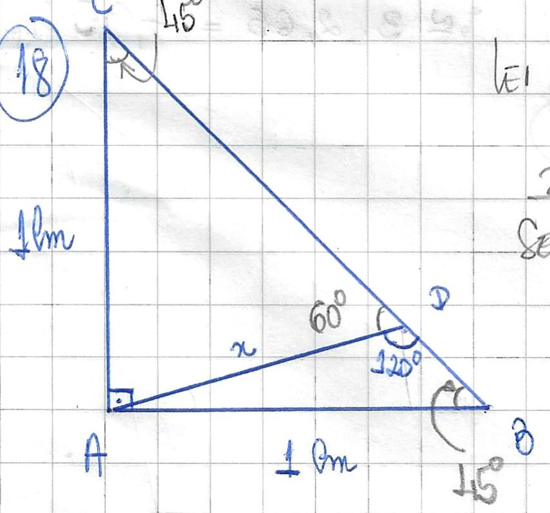
Relação Fundamental:

$$\operatorname{sen}^2\theta + \left(\frac{\sqrt{2}}{4}\right)^2 = 1 \Rightarrow \operatorname{sen}^2\theta = 1 - \frac{\cancel{2}}{\cancel{16}_{8}} \Rightarrow \operatorname{sen}^2\theta = \frac{7}{8}$$

$$\Rightarrow \operatorname{sen}\theta = \sqrt{\frac{7}{8}} = \frac{\sqrt{7}}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$\therefore \boxed{\operatorname{sen}\theta = \frac{\sqrt{14}}{4}}$$

18



LEI DOS SENOS ΔACD:

$$\frac{x}{\operatorname{sen}45^\circ} = \frac{1}{\operatorname{sen}60^\circ} \Leftrightarrow x \operatorname{sen}60^\circ = 1 \operatorname{sen}45^\circ$$
$$\Leftrightarrow x \cdot \frac{\sqrt{3}}{\cancel{2}} = \frac{\sqrt{2}}{\cancel{2}}$$
$$\Leftrightarrow x = \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{6}}{3}\ \text{cm}$$

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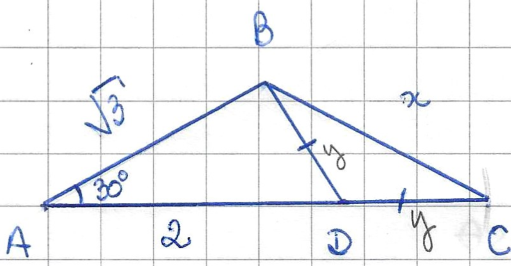


Lei dos Cossenos ΔABD:

$$y^2 = (\sqrt{3})^2 + 2^2 - 2 . \sqrt{3} . 2 \cos 30°$$
$$y^2 = 3 + 4 - \cancel{4}^2\sqrt{3} . \frac{\sqrt{3}}{\cancel{2}}$$
$$y^2 = 7 - 6 \Rightarrow y = 1 \text{ cm}$$
$$\Rightarrow \overline{AC} = 3 \text{ cm}.$$

Lei dos Cossenos ΔABC:

$$x^2 = (\sqrt{3})^2 + 3^2 - 2\sqrt{3} . 3 . \cos 30°$$
$$x^2 = 3 + 9 - \cancel{6}^3\sqrt{3} . \frac{\sqrt{3}}{\cancel{2}}$$
$$x^2 = 3 + \cancel{9} - \cancel{9} \quad \therefore \boxed{x = \sqrt{3} \text{ cm}}$$

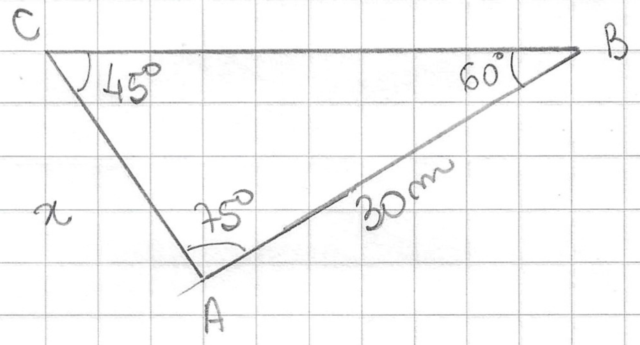
20 $B\hat{A}C = 60°$ ($\hat{A}$ngulo inscrito de $B\hat{O}C = 120°$)

Lei dos Cossenos:

$$\overline{BC}^2 = 6^2 + 9^2 - 2 . 6 . 9 . \cos 60°$$
$$\overline{BC}^2 = 36 + 81 - \cancel{2} . 54 . \frac{1}{\cancel{2}}$$
$$\overline{BC}^2 = 63 \Rightarrow \overline{BC} = \sqrt{63} = \sqrt{9 . 7} \quad \therefore \overline{BC} = 3\sqrt{7} \text{ cm}$$
$$\cong 3 . 2{,}65 = 7{,}95 \text{ cm}$$

$$2P: 6 + 9 + 7{,}95 = 22{,}95 \text{ cm} \quad \text{e}$$

21



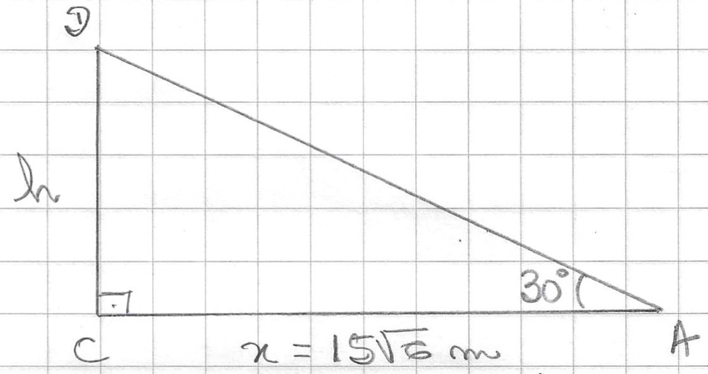
LEI DOS SENOS:

$$\frac{x}{\operatorname{sen} 60°} = \frac{30}{\operatorname{sen} 45°}$$

$$x \cdot \frac{\sqrt{2}}{\cancel{2}} = 30 \cdot \frac{\sqrt{3}}{\cancel{2}}$$

$$x = \frac{30\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{30\sqrt{6}}{2}$$

$$x = 15\sqrt{6}\ m$$



$$tg\,30° = \frac{h}{15\sqrt{6}} = \frac{\sqrt{3}}{3}$$

$$\Rightarrow h = \frac{\overset{5}{\cancel{15}}\sqrt{6} \cdot \sqrt{3}}{\cancel{3}}$$

$$\Rightarrow h = 5\sqrt{18} \Rightarrow h = 5\sqrt{9 \cdot 2} = 5 \cdot 3\sqrt{2}$$

$$\Rightarrow h = 15\sqrt{2}\ m$$

$$\therefore \boxed{\frac{h}{\sqrt{2}} = 15}$$