Lista 10 - Mat I

1 Se a parábola só tem 1 ponto de interseção com o eixo x significa que só tem uma raiz. Logo, $\Delta = 0$.

$y = x^2 - mx + (m-1)$.

$a = 1$, $b = -m$, $c = m-1$

$\boxed{\Delta = 0}$

$(-m)^2 - 4.1.(m-1) = 0$

$m^2 - 4(m-1) = 0$

$\underbrace{m^2 - 4m + 4 = 0}$

$(m-2)^2 = 0$

$\downarrow$

$m - 2 = 0$

$\boxed{m = 2}$

Substituindo $m = 2$ na função, temos:

$\boxed{y = x^2 - 2x + 1}$

$x = 2 \rightarrow y = 2^2 - 2.2 + 1$

$y = 4 - 4 + 1$

$\boxed{y = 1}$

letra d

2 Os 3 pontos dados pertencem à uma parábola

$y = ax^2 + bx + c$

$(1,3) \xRightarrow{x=1} a \cdot 1^2 + b.1 + c = 3$

$\boxed{a + b + c = 3}$

$(2,5) \xRightarrow{x=2} a \cdot 2^2 + b.2 + c = 5$

$\boxed{4a + 2b + c = 5}$

$(3,1) \xRightarrow{x=3} a \cdot 3^2 + b\,3 + c = 1$

$\boxed{9a + 3b + c = 1}$

Sistema com 3 equações:

$$\begin{cases} a + b + c = 3 \rightsquigarrow c = 3 - a - b \\ 4a + 2b + c = 5 \\ 9a + 3b + c = 1 \end{cases}$$

Substituindo $c = 3 - a - b$

$$\begin{cases} 4a + 2b + 3 - a - b = 5 \\ 9a + 3b + 3 - a - b = 1 \end{cases}$$

$$\begin{cases} 3a + b = 2 \quad .(-2) \\ 8a + 2b = -2 \end{cases}$$

$$\begin{cases} -6a - \cancel{2b} = -4 \\ 8a + \cancel{2b} = -2 \end{cases} +$$

$2a = -6$

$\boxed{a = -3}$

$3a+b=+2$

$3.(-3)+b=2$

$-9+b=2$

$\boxed{b=11}$

$c=3-a-b$

$c=3-(-3)-11$

$c=3+3-11$

$\boxed{c=-5}$

$y=-3x^2+11x-5$

Para $x=2,5$, temos:

$y=-3.2,5^2+\underbrace{11.2,5}-5$

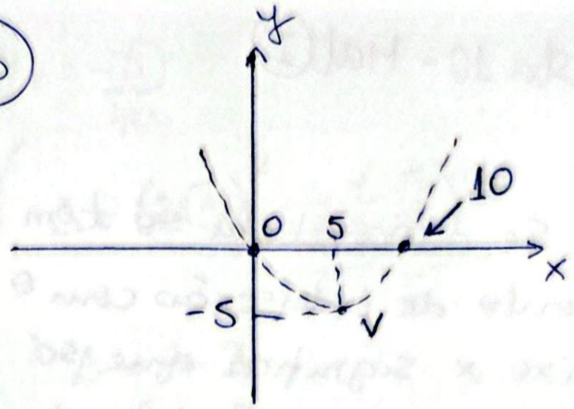
$y=\underbrace{-3.6,25}+27,5-5$

$y=-18,75+\underbrace{27,5-5}$

$y=-18,75+22,5$

$y=3,75$ letra a

3



raizes: 0 e 10 (simetria)

$(0,0) \Rightarrow a.0^2+b.0+c=0$

$\boxed{c=0}$

$(10,0) \Rightarrow a.10^2+b.10+\cancel{c}=0$

$100a+10b=0 \quad \div 10$

$\boxed{10a+b=0}$

$(5,-5) \Rightarrow a.5^2+b.5+\cancel{c}=-5$

$25a+5b=-5 \quad \div 5$

$\boxed{5a+b=-1}$

Sistema:

$\begin{cases} 10a+b=0 \\ 5a+b=-1 \quad .(-1) \end{cases}$

$\begin{cases} 10a+\cancel{b}=0 \\ -5a-\cancel{b}=1 \end{cases}$

$5a=1$

$\boxed{a=\frac{1}{5}}$

$5a+b=-1$

$5\cdot\frac{1}{5}+b=-1$

$1+b=-1$

$\boxed{b=-2}$

$y=\frac{1}{5}x^2-2x$

letra a

(4)

$$C = 2510 - 100n + n^2$$

função quadrática

$\boxed{a = 1}$, $b = -100$, $C = 2510$

$\oplus$

tem valor mínimo

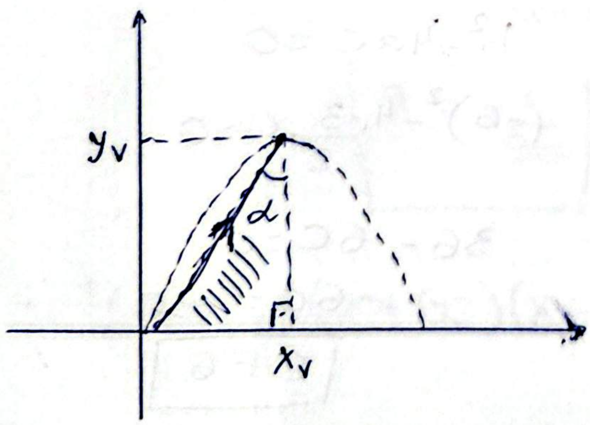
n^{o} de unidades para se obter o mínimo é o x_v.

$$x_v = -\frac{b}{2a} = -\frac{(-100)}{2 \cdot 1} = \frac{100}{2}$$

$$x_v = \boxed{50}$$

(5)

$$f(x) = \left(-\frac{\sqrt{3}}{3}\right)x^2 + 2\sqrt{3}\,x$$



$$x_v = -\frac{b}{2a} = -\frac{2\sqrt{3}}{2 \cdot \left(-\frac{\sqrt{3}}{3}\right)} = \frac{-2\sqrt{3}}{\frac{-2\sqrt{3}}{3}}$$

$$= \cancel{-2\sqrt{3}} \cdot \frac{3}{\cancel{-2\sqrt{3}}} = \boxed{3}$$

$$y_v = \frac{-\Delta}{4a}$$

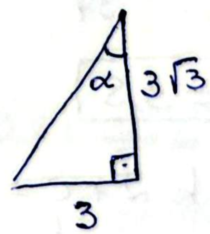
$$\Delta = (2\sqrt{3})^2 - \underbrace{4 \cdot \left(-\frac{\sqrt{3}}{3}\right) \cdot 0}_{0}$$

$$\Delta = 4 \cdot 3$$
$$\Delta = 12$$

$$y_v = \frac{-12}{4 \cdot \left(-\frac{\sqrt{3}}{3}\right)} = \frac{-12}{\frac{-4\sqrt{3}}{3}}$$

$$= -\cancel{12}^{\,3} \cdot \frac{3}{\cancel{-4}\sqrt{3}}$$

$$= \frac{9}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{9\sqrt{3}}{3} = 3\sqrt{3}$$



$$\operatorname{tg}\alpha = \frac{\cancel{3}}{\cancel{3}\sqrt{3}}$$

$$\operatorname{tg}\alpha = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\operatorname{tg}\alpha = \frac{\sqrt{3}}{3}$$

$$\alpha = 30^{\circ}$$

letra (a)

⑥

medições: (intervalo de 1h)

temp(°C)

$1^a \to t=0 \Rightarrow 37°$

$2^a \to t=1 \Rightarrow 40{,}5°$

$3^a \to t=2 \Rightarrow 39°$

$$f(x)=ax^2+bx+c$$

$$\downarrow$$

$$T(t)=a\cdot t^2+b\cdot t+c$$

$$(0,37) \to a\cdot 0^2+b\cdot 0+c=37$$

$$\boxed{c=37}$$

$$(1,\ 40{,}5) \to a\cdot 1^2+b\cdot 1+\underset{\substack{\downarrow \\ 37}}{c}=40{,}5$$

$$\boxed{a+b=3{,}5}$$

$$(2,39) \to a\cdot 2^2+b\cdot 2+\underset{\substack{\downarrow \\ 37}}{c}=39$$

$$4a+2b=2 \quad \div 2$$

$$\boxed{2a+b=1}$$

$$\begin{cases} a+b=3{,}5 & \cdot(-1) \\ 2a+b=1 \end{cases}$$

$$\begin{cases} -a-\cancel{b}=-3{,}5 \\ 2a+\cancel{b}=1 \end{cases} \quad +$$

$$\boxed{a=-2{,}5}$$

$$a+b=3{,}5$$

$$\downarrow$$

$$-2{,}5+b=3{,}5$$

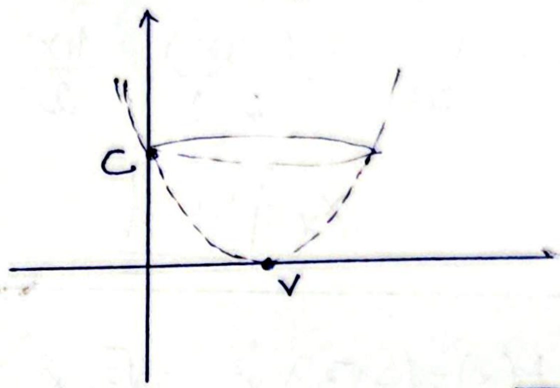
$$b=3{,}5+2{,}5$$

$$\boxed{b=6}$$

$$T(t)=-2{,}5t^2+6t+37$$

letra c

⑦ $f(x)=\left(\frac{3}{2}\right)x^2-6x+c$



só tem uma raiz $\Rightarrow \boxed{\Delta=0}$

$$b^2-4ac=0$$

$$(-6)^2-4\cdot\frac{3}{2}\cdot c=0$$

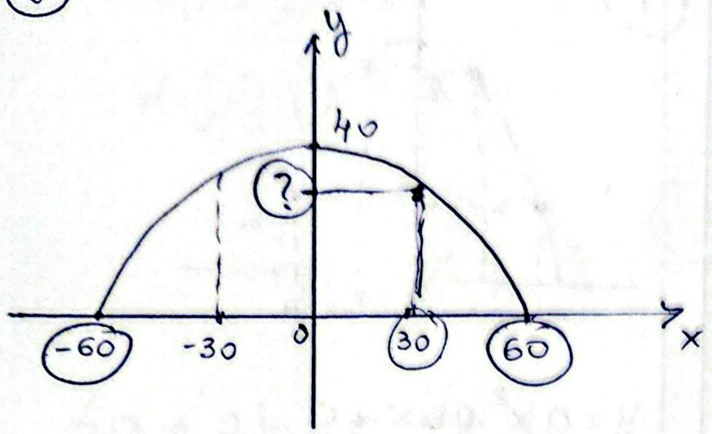
$$36-6c=0$$

$$-6c=-36$$

$$\boxed{c=6}$$

letra e

8



forma fatorada

$y = a \cdot (x - r_1)(x - r_2)$

$y = a\,(x+60)(x-60)$

?

$x = 0 \longrightarrow a \cdot (0+60).(0-60) = 40$

$\underbrace{a \cdot 60(-60)} = 40$

$-3600a = 40$

$a = -\frac{4\cancel{0}}{360\cancel{0}}$

$\boxed{a = -\frac{1}{90}}$

$y = -\frac{1}{90}.(x+60)(x-60)$

Para $x = 30$, temos:

$y = -\frac{1}{90}.(30+60)(30-60)$

$= -\frac{1}{\cancel{90}}.\cancel{90}.(-30) = 30$

Soma:

$2x(40 + 30 + 30)$

$2 \times 100 = \underline{200\ m}$

letra e

OU

use os pares ordenados:

$(0,40) \rightarrow a \cdot 0^2 + b \cdot 0 + c = 40$

$\boxed{c = 40}$

$(60,0) \rightarrow a \cdot 60^2 + b \cdot 60 + c = 0$

$3600a + 60b + 40 = 0$

$\div 20$

$\boxed{180a + 3b + 2 = 0}$

$(-60,0) \longrightarrow a \cdot (-60)^2 + b \cdot (-60) + c = 0$

$3600a - 60b + 40 = 0$

$\div 20$

$\boxed{180a - 3b + 2 = 0}$

Sistema:

$\begin{cases} 180a + \cancel{3b} + 2 = 0 \\ 180a - \cancel{3b} + 2 = 0 \end{cases}$ +

$360a + 4 = 0$

$360a = -4$

$a = \frac{-4}{360} = \boxed{-\frac{1}{90}}$

$$180a + 3b + 2 = 0$$

$$\underbrace{180\left(\frac{-1}{90}\right)} + 3b + 2 = 0$$

$$-2 + 3b + 2 = 0$$

$$3b = 0$$

$$\boxed{b = 0}$$

$$\boxed{y = -\frac{1}{90}x^2 + 40}$$

Para $x = 30$, temos:

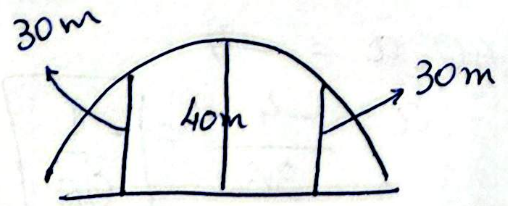
$$y = -\frac{1}{90} \cdot 30^2 + 40$$

$$y = \underbrace{-\frac{1}{90} \cdot 900} + 40$$

$$y = -10 + 40$$

$$\boxed{y = 30m}$$

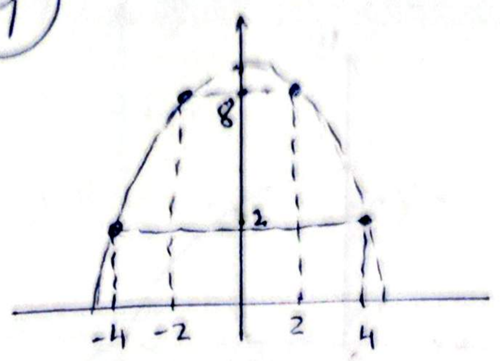
(a medida na posição de $x = 30$ dá 30m de altura)



Em cada lado: $40 + 30 + 30 = 100m$

Total: 200m

9



$$y = ax^2 + bx + c$$

temos 4 pontos:

$(2,8) \Rightarrow a \cdot 2^2 + b \cdot 2 + c = 8$

$(-2,8) \Rightarrow a \cdot (-2)^2 + b \cdot (-2) + c = 8$

$(4,2) \Rightarrow a \cdot 4^2 + b \cdot 4 + c = 2$

$(-4,2) \Rightarrow a \cdot (-4)^2 + b \cdot (-4) + c = 2$

$$\begin{cases} 4a + \cancel{2b} + c = 8 \\ 4a - \cancel{2b} + c = 8 \end{cases} \quad (+)$$

$$\boxed{8a + 2c = 16} \quad (\div 2)$$

$$\begin{cases} 16a + \cancel{4b} + c = 2 \\ 16a - \cancel{4b} + c = 2 \end{cases} \quad (+)$$

$$\boxed{32a + 2c = 4} \quad (\div 2)$$

$$\begin{cases} 4a + c = 8 \quad \cdot(-1) \\ 16a + c = 2 \end{cases}$$

$$\begin{cases} -4a - \cancel{c} = -8 \\ 16a + \cancel{c} = 2 \end{cases}$$

$$12a = -6$$

$$a = \frac{-6}{12} = -\frac{1}{2}$$

$4a + c = 8$

$4 \cdot \left(-\frac{1}{2}\right) + c = 8$

$-2 + c = 8$

$\boxed{c = 10}$

$4a + 2b + c = 8$

$4 \cdot \left(-\frac{1}{2}\right) + 2b + 10 = 8$

$-2 + 2b + 10 = 8$

$2b = 8 + 2 - 10$

$2b = 0$

$\boxed{b = 0}$

$$\boxed{y = -\frac{1}{2}x^2 + 10}$$

$a + b + c = ?$

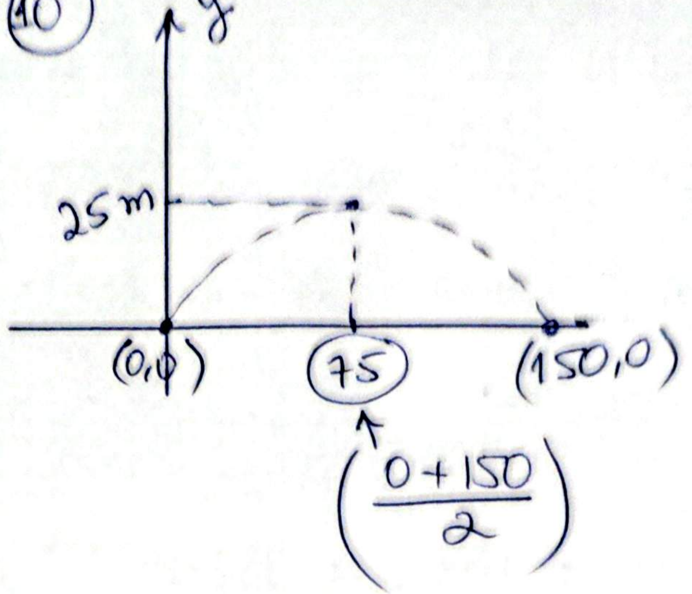
$-\frac{1}{2} + 0 + 10$

$-0{,}5 + 10$

$9{,}5$

letra d

10



Temos 3 pontos:

$(0,0) \Rightarrow a \cdot 0^2 + b \cdot 0 + c = 0$

$\boxed{c = 0}$

$(150,0) \Rightarrow a \cdot 150^2 + b \cdot 150 + \cancel{c}^{0} = 0$

$\div 150$

$\boxed{150a + b = 0}$

$(75,25) \Rightarrow a \cdot 75^2 + b \cdot 75 + \cancel{c}^{0} = 25$

$5625a + 75b = 25$

$\div 25$

$\boxed{225a + 3b = 1}$

Sistema:

$$\begin{cases} 150a + b = 0 & \cdot(-3) \\ 225a + 3b = 1 \end{cases}$$

$$\begin{cases} -450a - \cancel{3b} = 0 \\ 225a + \cancel{3b} = 1 \end{cases} \; (+)$$

$-225a = 1$

$a = -\frac{1}{225}$

$150a + b = 0$

$150 \cdot \left(-\frac{1}{225}\right) + b = 0$

$-\frac{150}{225} + b = 0$

$b = \frac{150}{225} \begin{matrix} \div 25 \\ \div 25 \end{matrix}$

$b = \frac{6}{9} \begin{matrix} \div 3 \\ \div 3 \end{matrix}$

$\boxed{b = \frac{2}{3}}$

$\boxed{y = -\frac{1}{225}x^2 + \frac{2}{3}x}$

Para encontrar a opção correta, vamos fazer o mmc(225, 3) = 225

$\frac{y}{1} = -\frac{1x^2}{225} + \frac{2x}{3}$ ($225/1$, $225/1$, $3/75$)

$225y = -x^2 + 150x$

$\boxed{225y = 150x - x^2}$

letra e