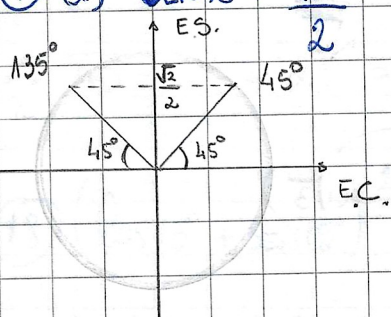


MATERNÁTICA II - LISTA 10

1) a) $\text{sen } x = \frac{\sqrt{2}}{2}$



$\text{sen } x > 0 \Rightarrow x \in 1^\circ \text{ ou } 2^\circ \text{ QUAD.}$

$x = \frac{\pi}{4} \text{ ou } x = \frac{3\pi}{4}$

$S = \left\{ \frac{\pi}{4}, \frac{3\pi}{4} \right\}$

b) $\cos^2 x = 1 \Rightarrow \cos x = \pm 1 \Rightarrow x = 0 \text{ ou } x = \pi$

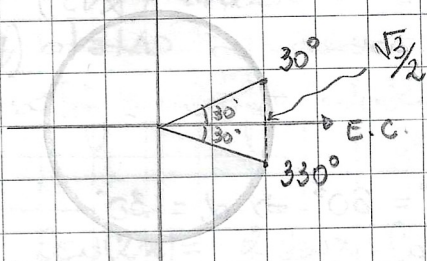
$S = \{0, \pi\}$

c) $2 \cos x - \sqrt{3} = 0$

$1^\circ \text{ ou } 4^\circ \text{ QUAD.}$

$2 \cos x = \sqrt{3} \Rightarrow \cos x = \frac{\sqrt{3}}{2}$

$\Rightarrow x = \frac{\pi}{6} \text{ ou } x = \frac{11\pi}{6}$



$S = \left\{ \frac{\pi}{6}, \frac{11\pi}{6} \right\}$

d) $\cos^2 x + \cos x = 0$, seja $\cos x = y$

$y^2 + y = 0$

EQUAÇÃO DO
2º GRAU.

RESOLVA DO JEITO
QUE QUISER!

$\Rightarrow y(y+1) = 0 \Rightarrow \begin{cases} y_1 = 0 \\ y_2 = -1 \end{cases} \Rightarrow \begin{cases} \cos x = 0 \\ \cos x = -1 \end{cases}$

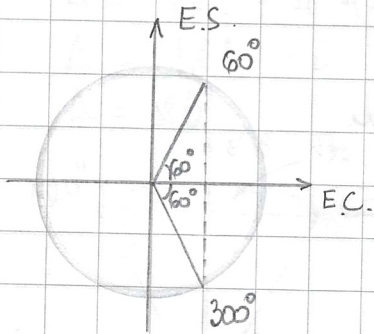
$\Rightarrow \begin{cases} x = \pi/2, x = 3\pi/2 \\ x = \pi \end{cases}$

$S = \left\{ \pi/2, \pi, 3\pi/2 \right\}$

e) $2 \cos^2 x = \cos x$; $\cos x = y$

$$2y^2 = y \Rightarrow 2y^2 - y = 0 \Rightarrow y(2y - 1) = 0 \Rightarrow \begin{cases} y_1 = 0 \\ y_2 = 1/2 \end{cases}$$

$$\Rightarrow \begin{cases} \cos x = 0 \\ \cos x = 1/2 \end{cases} \Rightarrow \begin{cases} x = \pi, x = 3\pi/2 \\ x = \pi/3, x = 5\pi/3 \end{cases}$$



$$S = \left\{ \frac{\pi}{3}, \pi, \frac{3\pi}{2}, \frac{5\pi}{3} \right\}$$

f) $2 \sin^2 x + 3 \sin x - 2 = 0$; $\sin x = y$

$$2y^2 + 3y - 2 = 0 \rightarrow y = \frac{-3 \pm \sqrt{25}}{2 \cdot 2} = \frac{-3 \pm 5}{4}$$

$$\Delta = 3^2 - 4 \cdot 2 \cdot (-2)$$

$$\Delta = 9 + 16$$

$$\Delta = 25$$

$$\begin{cases} y_1 = -2 \\ y_2 = \frac{1}{2} \end{cases}$$

$$\begin{aligned} y_1 &= \frac{-3-5}{4} = -\frac{8}{4} \\ y_2 &= \frac{-3+5}{4} = \frac{2}{4} \end{aligned}$$

① $\sin x = -2$
NÃO TEM SOLUÇÃO
Pois $-1 \leq \sin x \leq 1$

② $\sin x = 1/2$ 30° ou 150°
 $S = \left\{ \frac{\pi}{6}, \frac{5\pi}{6} \right\}$

g) $2 \cos^2 x + 11 \cos x = -5$; $\cos x = y$

$$2y^2 + 11y + 5 = 0 \rightarrow y = \frac{-11 \pm 9}{4} \rightarrow \begin{cases} y_1 = \frac{-11-9}{4} = -\frac{20}{4} = -5 \\ y_2 = \frac{-11+9}{4} = -\frac{2}{4} = -\frac{1}{2} \end{cases}$$

$$\Delta = 121 - 4 \cdot 2 \cdot 5$$

$$\Delta = 121 - 40$$

$$\Delta = 81$$

$\cos x = -5$ NÃO TEM SOLUÇÃO
 $\cos x = -1/2$

120° ou 240°

$$S = \left\{ 2\pi/3, 4\pi/3 \right\}$$

$$h) 4 \sin x - \sqrt{3} = 2 \sin x$$

$$4 \sin x - 2 \sin x = \sqrt{3}$$

$$2 \sin x = \sqrt{3}$$

$$\sin x = \frac{\sqrt{3}}{2}$$

60° ou 120°

$$S = \left\{ \pi/3, 2\pi/3 \right\}$$

$$i) \sin(2x) = \frac{1}{2}$$

30° ou 150°

$$\Rightarrow 2x = \pi/6 \Rightarrow x = \frac{\pi}{12} \text{ ou } 2x = \frac{5\pi}{6} \Rightarrow x = \frac{5\pi}{12}$$

$$2x = \frac{5\pi}{6} \Rightarrow x = \frac{5\pi}{12}$$

$$S = \left\{ \pi/12, 5\pi/12 \right\}$$

$$j) 2 \cos^2 x = 1 - \sin x \Rightarrow 2(1 - \sin^2 x) = 1 - \sin x, \sin x = y$$

OBS:

$$\sin^2 x + \cos^2 x = 1$$

$$\cos^2 x = 1 - \sin^2 x$$

$$2(1 - y^2) = 1 - y \Rightarrow 2 - 2y^2 = 1 - y$$

$$\Rightarrow 2y^2 - y - 1 = 0$$

$$\Delta = 1 - 4 \cdot 2 \cdot (-1)$$

$$\Delta = 9$$

$$y = \frac{1 \pm 3}{4}$$

$$y_1 = -2/4 = -1/2$$

$$y_2 = 1$$

$$\Rightarrow \begin{cases} \sin x = -1/2 \\ \sin x = 1 \end{cases}$$

210° ou 330°

90°

$$S = \left\{ \pi/2, \frac{7\pi}{6}, \frac{11\pi}{6} \right\}$$

$$K) 4 \operatorname{sen}^4 x - 11 \operatorname{sen}^2 x + 6 = 0$$

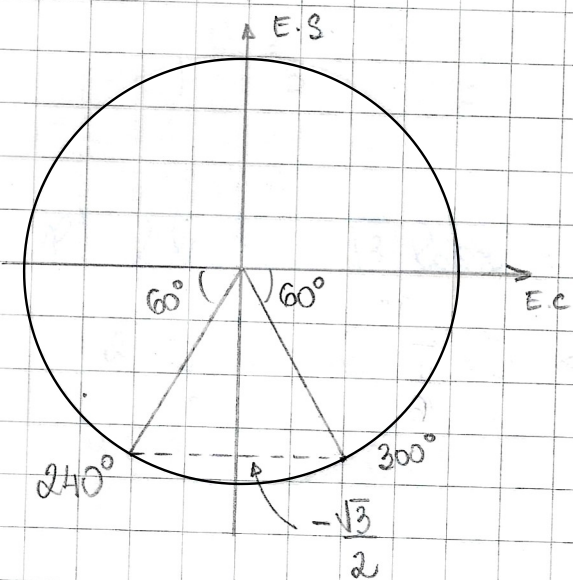
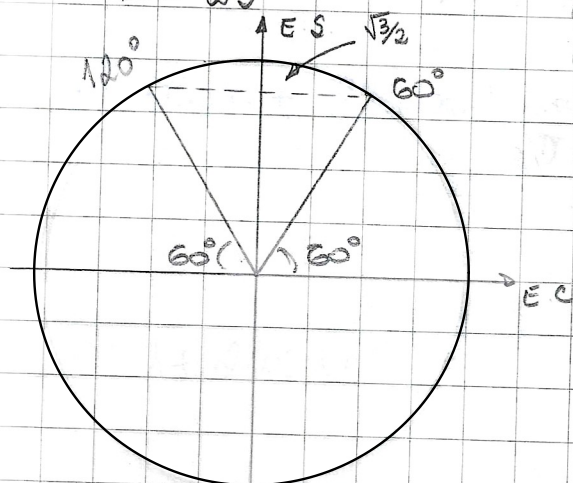
$$\operatorname{sen}^2 x = y$$

$$4y^2 - 11y + 6 = 0$$

$$\Delta = 121 - 4 \cdot 4 \cdot 6$$

$$\Delta = 121 - 96$$

$$\Delta = 25$$



$$y = \frac{11 \pm 5}{8}$$

$$y_1 = 2$$

$$y_2 = \frac{6}{8} = \frac{3}{4}$$

$$\Rightarrow \operatorname{sen}^2 x = 2 \Rightarrow \operatorname{sen} x = \pm \sqrt{2}$$

NAO TEM SOLUÇÃO, Pois $\sqrt{2} > 1$.

$$\operatorname{sen}^2 x = \frac{3}{4} \Rightarrow \operatorname{sen} x = \frac{\sqrt{3}}{2} \text{ ou } \operatorname{sen} x = -\frac{\sqrt{3}}{2}$$

$\operatorname{sen}(x) = \frac{\sqrt{3}}{2}$

$$S = \left\{ \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3} \right\}$$

$$l) 2 \sin x - \frac{1}{\sin x} = 1, \quad \sin x = y$$

$$\frac{2y}{1/y} - \frac{1}{y/1} = 1 \Rightarrow \frac{2y^2 - 1}{y} = 1 \Rightarrow 2y^2 - 1 = y$$

$$2y^2 - y - 1 = 0$$

$$\Delta = 1 - 4 \cdot 2 \cdot (-1)$$

$$\Delta = 1 + 8$$

$$\Delta = 9$$

$$y = \frac{1 \pm 3}{4}$$

$$y_1 = 1$$

$$y_2 = -\frac{1}{2}$$

$$\sin x = 1 \Rightarrow x = \pi/2$$

$$\sin x = -1/2 \Rightarrow x = \text{ou}$$

210° ou 330°

$$S = \left\{ \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6} \right\}$$

$$m) 2 \sin x - \cos x - \cos x = 0$$

$$\cos x (2 \sin x - 1) = 0$$

$$\Rightarrow \cos x = 0 \Rightarrow x = \pi/2 \text{ ou } x = 3\pi/2$$

ou

$$2 \sin x - 1 = 0 \Rightarrow \sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6} \text{ ou } x = \frac{5\pi}{6}$$

30° ou 150°

$$S = \left\{ \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6} \right\}$$

$$n) \sin^2 x - \sin x = 0, \sin x = y$$

$$y^2 - y = 0$$

$$y(y-1) = 0 \Rightarrow y = 0 \Rightarrow \sin x = 0 \Rightarrow x = 0, x = \pi$$

$$y = 1 \Rightarrow \sin x = 1 \Rightarrow x = \pi/2$$

$$S = \{0, \pi/2, \pi\}$$

$$p) \sin x + 2 \sin x \cdot \cos x = 0$$

$$\sin x (1 + 2 \cos x) = 0$$

$$\sin x = 0 \Rightarrow x = 0 \text{ ou } x = \pi$$

$$1 + 2 \cos x = 0 \Rightarrow \cos x = -\frac{1}{2} \Rightarrow x = \frac{2\pi}{3} \text{ ou } x = \frac{4\pi}{3}$$

$$120^\circ \text{ ou } 240^\circ$$

$$S = \{0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}\}$$

$$r) 2 \cos^2 x - \sqrt{3} \cos x = 0, \cos x = y$$

$$2y^2 - \sqrt{3}y = 0 \Rightarrow y_1 = 0$$

$$\Rightarrow \cos x = 0 \Rightarrow x = \pi \text{ ou } x = 3\pi/2$$

$$y(2y - \sqrt{3}) = 0 \Rightarrow y_2 = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \cos x = \frac{\sqrt{3}}{2} \Rightarrow x = \frac{\pi}{6} \text{ ou } x = \frac{11\pi}{6}$$

$$30^\circ \text{ ou } 330^\circ$$

$$S = \{\frac{\pi}{6}, \pi, \frac{3\pi}{2}, \frac{11\pi}{6}\}$$

$$g) 4\cos x + 3\sec x = 8, \cos x = y$$

$$\frac{4y}{\cancel{1}} + \frac{3}{\cancel{y}} = 8 \Rightarrow \frac{4y^2 + 3}{y} = 8 \Rightarrow 4y^2 + 3 = 8y$$

$$4y^2 - 8y + 3 = 0 \Rightarrow y = \frac{8 \pm 4}{8} \begin{cases} y_1 = \frac{12}{8} = \frac{3}{2} \\ y_2 = \frac{4}{8} = \frac{1}{2} \end{cases}$$

$$\Delta = 64 - 4 \cdot 4 \cdot 3$$

$$\Delta = 64 - 48$$

$$\Delta = 16$$

$$\cos x = \frac{3}{2} \text{ Não tem solução, } \frac{3}{2} > 1$$

$$\cos x = \frac{1}{2} \Rightarrow x = \frac{\pi}{3} \text{ ou } x = \frac{5\pi}{3}$$

$$60^\circ \text{ ou } 300^\circ$$

$$S = \left\{ \frac{\pi}{3}, \frac{5\pi}{3} \right\}$$

$$x) \sec^2 x = 1 + \tan x$$

$$1 + \tan^2 x = 1 + \tan x$$

$$\tan^2 x - \tan x = 0, \tan x = y$$

$$y^2 - y = 0 \Rightarrow y(y-1) = 0$$

$$y = 0 \Rightarrow \tan x = 0 \Rightarrow x = 0 \text{ ou } \pi$$

$$y = 1 \Rightarrow \tan x = 1 \Rightarrow x = \frac{\pi}{4} \text{ ou } \frac{5\pi}{4}$$

$$45^\circ \text{ ou } 225^\circ$$

$$S = \left\{ 0, \frac{\pi}{4}, \pi, \frac{5\pi}{4} \right\}$$

$$b) 1 - 2 \sin^2 x = \cos^2 x$$

$$\left\{ \begin{array}{l} 1 - \sin^2 x, \quad \sin x = y \end{array} \right.$$

$$1 - 2y^2 = 1 - y^2 \Rightarrow -y^2 = 0 \Rightarrow y = 0$$

$$\Rightarrow \sin x = 0, \quad x = 0 \text{ ou } x = \pi$$

$$S = \{0, \pi\}$$

$$t) 3(\tan^2 x - 1) = 2\sqrt{3} \tan x, \quad \tan x = y$$

$$3(y^2 - 1) = 2\sqrt{3}y$$

$$3y^2 - 3 = 2\sqrt{3}y \Rightarrow 3y^2 - 2\sqrt{3}y - 3 = 0$$

$$\sqrt{16 \cdot 3} = 4\sqrt{3}$$

$$\Delta = (-2\sqrt{3})^2 - 4 \cdot 3 \cdot (-3) \Rightarrow y = \frac{2\sqrt{3} \pm \sqrt{48}}{6}$$

$$\Delta = 4 \cdot 3 + 36$$

$$\Delta = 48$$

$$y = \frac{2\sqrt{3} \pm 4\sqrt{3}}{6}$$

$$y_1 = \frac{6\sqrt{3}}{6} = \sqrt{3}; \quad y_2 = \frac{-2\sqrt{3}}{6} = -\frac{\sqrt{3}}{3}$$

$$① \tan x = \sqrt{3} \Rightarrow x = \frac{\pi}{3} \text{ ou } x = \frac{4\pi}{3}$$

$$60^\circ \text{ ou } 240^\circ$$

$$② \tan x = -\frac{\sqrt{3}}{3} \Rightarrow x = \frac{5\pi}{6} \text{ ou } x = \frac{11\pi}{6}$$

$$150^\circ \text{ ou } 330^\circ$$

$$S = \left\{ \frac{\pi}{3}, \frac{5\pi}{6}, \frac{4\pi}{3}, \frac{11\pi}{6} \right\}$$

$$\textcircled{2} \quad \tan^2 x + \sec^2 x = 3 \cos^2 x$$

OBS:

$$\begin{aligned} \sec^2 x &= 1 + \tan^2 x \\ \Rightarrow \sec^2 x - 1 &= \tan^2 x \end{aligned}$$

$$\sec^2 x - 1 + 1 - \cos^2 x = 3 \cos^2 x$$

$$\frac{1}{\cos^2 x} - \cos^2 x = 3 \cos^2 x, \quad \cos x = y$$

$$\frac{1}{y^2} - y^2 = 3y^2 \Rightarrow \frac{1}{y^2} = 4y^2 \Rightarrow y^4 = \frac{1}{4}$$

$$\Rightarrow y = \pm \sqrt[4]{\frac{1}{4}} = \pm \frac{1}{\sqrt[4]{4}} = \pm \frac{1}{\sqrt{2} \sqrt{2}} \Rightarrow y = \pm \frac{\sqrt{2}}{2}$$

$$\sqrt[4]{4} = \sqrt[4]{2^2} = \sqrt{2}$$

$$\textcircled{1} \quad \cos x = \frac{\sqrt{2}}{2} \Rightarrow x = \frac{\pi}{4} \text{ ou } x = \frac{7\pi}{4}$$

$45^\circ \text{ ou } 315^\circ$

$$\textcircled{2} \quad \cos x = -\frac{\sqrt{2}}{2} \Rightarrow x = \frac{3\pi}{4} \text{ ou } x = \frac{5\pi}{4}$$

$135^\circ \text{ ou } 225^\circ$

$$\text{Somme: } \frac{\pi}{4} + \frac{7\pi}{4} + \frac{3\pi}{4} + \frac{5\pi}{4} = \frac{16\pi}{4} = 4\pi \quad \textcircled{A}$$

$$\textcircled{3} \quad 2 \operatorname{sen} \theta = 3 \operatorname{tg}^2 \theta$$

$$\frac{\operatorname{sen}^2 \theta}{\cos^2 \theta} \quad \swarrow \quad 1 - \operatorname{sen}^2 \theta$$

$$\operatorname{sen} \theta = y$$

$$2y = \frac{3 \cdot y^2}{1 - y^2} \Rightarrow 2y(1 - y^2) = 3y^2 \Rightarrow 2y - 2y^3 = 3y^2$$

$$\Rightarrow 2y^3 + 3y^2 - 2y = 0 \Rightarrow y(2y^2 + 3y - 2) = 0$$

$$\Rightarrow y_1 = 0 \quad \text{ou} \quad 2y^2 + 3y - 2 = 0$$

$$\Delta = 9 - 4 \cdot 2 \cdot (-2)$$

$$\Delta = 25$$

$$y = \frac{-3 \pm 5}{4}$$

$$\begin{cases} y_2 = -2 \\ y_3 = \frac{1}{2} \end{cases}$$

$\operatorname{sen} \theta = 0$ NÃO TEM SOLUÇÃO EM $0 < \theta < \pi/2$

$\operatorname{sen} \theta = -2$ NÃO TEM SOLUÇÃO NUNCA!

$$\operatorname{sen} \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6} \Rightarrow \cos \theta = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$\swarrow$
 30°

$$\textcircled{4} \quad 2 \cos^2 x - \operatorname{sen} x = 1 \Rightarrow 2(1 - \operatorname{sen}^2 x) - \operatorname{sen} x - 1 = 0, \quad \operatorname{sen} x = y$$

$$2(1 - y^2) - y - 1 = 0 \Rightarrow 2 - 2y^2 - y - 1 = 0$$

$$\Rightarrow 2y^2 + y - 1 = 0 \Rightarrow y = \frac{-1 \pm 3}{4} \quad \begin{cases} y_1 = -1 \Rightarrow \operatorname{sen} x = -1 \Rightarrow x = 3\pi/2 \\ y_2 = 1/2 \Rightarrow \operatorname{sen} x = 1/2 \Rightarrow x = \pi/6 \text{ ou } 5\pi/6 \end{cases}$$

$$\Delta = 1 - 4 \cdot 2 \cdot (-1)$$

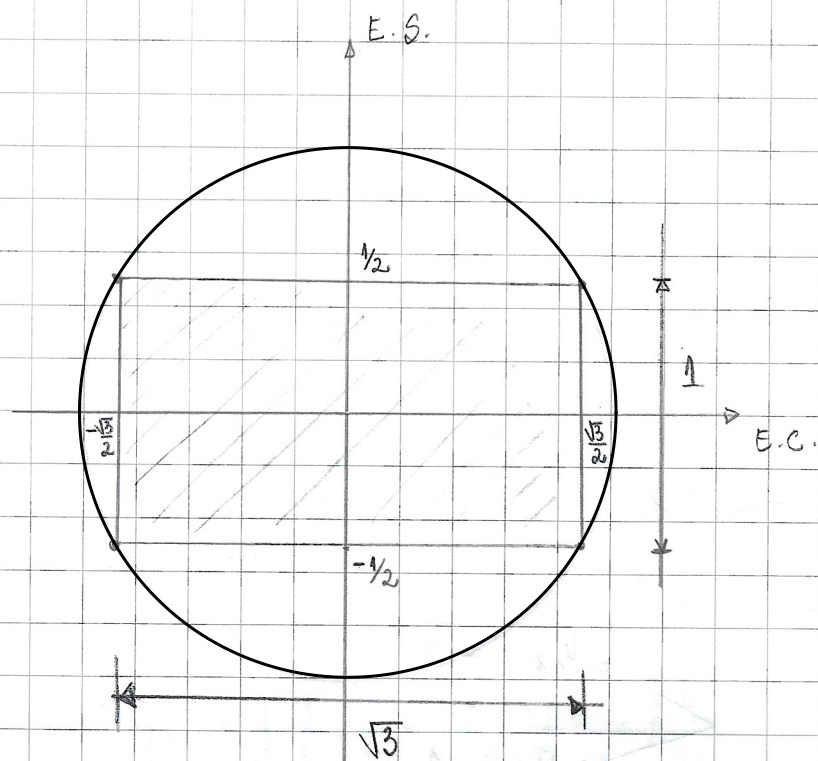
$$\Delta = 9$$

$30^\circ \text{ ou } 150^\circ$

$\textcircled{B}$

3 Soluções

$$\textcircled{5} \quad 4 \operatorname{sen}^2 x = 1 \Rightarrow \operatorname{sen}^2 x = \frac{1}{4} \Rightarrow \operatorname{sen} x = \pm \frac{1}{2}$$



$$A_{\text{RETÂNGULO}} = \sqrt{3} \cdot 1 = \sqrt{3}$$