MATEMÁTICA II - Lista 9

① a) $\text{sen}\,105° = \text{sen}(60°+45°) = \text{sen}\,60° \cdot \cos 45° + \text{sen}\,45° \cdot \cos 60°$

$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$

b) $\cos 105° = \cos(60°+45°) = \cos 60° \cos 45° - \text{sen}\,60° \cdot \text{sen}\,45°$

$= \frac{1}{2}\,\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}-\sqrt{6}}{4}$

c) $\text{tg}\,105° = \text{tg}(60°+45°) = \frac{\text{tg}\,60° + \text{tg}\,45°}{1 - \text{tg}\,60°\,\text{tg}\,45°} = \frac{\sqrt{3}+1}{1-\sqrt{3}\cdot 1}$

$= \frac{\sqrt{3}+1}{1-\sqrt{3}} \cdot \frac{(1+\sqrt{3})}{(1+\sqrt{3})} = \frac{3+2\sqrt{3}+1}{1-3} = \frac{4+2\sqrt{3}}{-2} \begin{array}{l} \div 2 \\ \div 2 \end{array} = -2-\sqrt{3}$

Racionalização do denominador

d) $\text{sen}\,15° = \text{sen}(60°-45°) = \text{sen}\,60° \cos 45° - \text{sen}\,45° \cos 60°$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$

e) $\cos 75° = \text{sen}\,15° = \frac{\sqrt{6}-\sqrt{2}}{4}$

$75° + 15° = 90°$

f) $\text{tg}\,15° = \text{tg}(60°-45°) = \frac{\text{tg}\,60° - \text{tg}\,45°}{1 + \text{tg}\,60°\,\text{tg}\,45°} = \frac{\sqrt{3}-1}{1+\sqrt{3}\cdot 1}$

$= \frac{\sqrt{3}-1}{\sqrt{3}+1} \cdot \frac{(\sqrt{3}-1)}{(\sqrt{3}-1)} = \frac{3-2\sqrt{3}+1}{3-1} = \frac{4-2\sqrt{3}}{2} \begin{array}{l} \div 2 \\ \div 2 \end{array} = 2-\sqrt{3}$

② $\cos 735° = \cos 15° = \cos(60° - 45°)$

$$\begin{array}{r|l} 735° & 360° \\ \hline 15° & 2 \end{array}$$

$= \cos 60° \cdot \cos 45° + \operatorname{sen} 60° \operatorname{sen} 45°$

$= \dfrac{1}{2} \cdot \dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{3}}{2} \cdot \dfrac{\sqrt{2}}{2} = \dfrac{\sqrt{2} + \sqrt{6}}{4}$

③ $\cos x = 0{,}8$

$\operatorname{sen}^2 x + \cos^2 x = 1$

$\operatorname{sen}^2 x + 0{,}8^2 = 1$

$\operatorname{sen}^2 x + 0{,}64 = 1$

$\operatorname{sen}^2 x = 1 - 0{,}64$

$\operatorname{sen}^2 x = 0{,}36$

$\operatorname{sen} x = \pm 0{,}6$

$0 < x < \pi/2$ (1º QUAD.)

$\operatorname{sen} x = 0{,}6$

$\operatorname{sen} 2x = 2 \operatorname{sen} x \cdot \cos x$

$= 2 \cdot 0{,}6 \cdot 0{,}8$

$= 0{,}96$ ©

④ $\operatorname{sen} x = \dfrac{4}{5}$

$\operatorname{sen}^2 x + \cos^2 x = 1$

$\left(\dfrac{4}{5}\right)^2 + \cos^2 x = 1$

$\dfrac{16}{25} + \cos^2 x = 1$

$\cos^2 x = 1 - \dfrac{16}{25}$

$\cos^2 x = \dfrac{9}{25} \Rightarrow \cos x = \pm \dfrac{3}{5}$

$tg\,x < 0 \Rightarrow \cos x = -\dfrac{3}{5}$

$tg\,x = \dfrac{\frac{4}{5}}{-\frac{3}{5}} = \dfrac{4}{\cancel{5}} \cdot \left(-\dfrac{\cancel{5}}{3}\right) =$

Segue que $tg\,2x = \dfrac{2\,tg\,x}{1 - tg^2 x} = \dfrac{2 \cdot \left(-\frac{4}{3}\right)}{1 - \left(-\frac{4}{3}\right)^2} = \dfrac{-\frac{8}{3}}{1 - \frac{16}{9}}$

$= \dfrac{-\frac{8}{3}}{-\frac{7}{9}} = -\dfrac{8}{\cancel{3}} \cdot \left(-\dfrac{\cancel{9}^{3}}{7}\right) = \dfrac{24}{7}$

ⓐ

⑤ $\cot g x + \operatorname{tg} x = 3$

$$\frac{\cos x}{\operatorname{sen} x_{(\cos x)}} + \frac{\operatorname{sen} x}{\cos x_{(\operatorname{sen} x)}} = 3 \Rightarrow \frac{\overbrace{\cos^2 x + \operatorname{sen}^2 x}^{1}}{\operatorname{sen} x \cos x} = 3$$

$$\Rightarrow \frac{1}{\operatorname{sen} x \cos x} = 3 \Rightarrow \operatorname{sen} x \cos x = \frac{1}{3} \quad (\times 2)$$

$$\Rightarrow \underbrace{2 \operatorname{sen} x \cos x}_{\operatorname{sen}(2x)} = \frac{2}{3} \quad \therefore \boxed{\operatorname{sen}(2x) = \frac{2}{3}} \quad \text{d}$$

⑥ $(\operatorname{tg} 10° + \cot g 10°) \cdot \operatorname{sen} 20°$

$$\underbrace{\frac{1}{\operatorname{sen} 10° \cdot \cos 10°}} \cdot \operatorname{sen} 20° = \frac{1}{\cancel{\operatorname{sen} 10°} \cdot \cancel{\cos 10°}} \cdot 2 \cancel{\operatorname{sen} 10°} \cdot \cancel{\cos 10°} = 2 \quad \text{c}$$

CONFORME EXERCÍCIO ANTERIOR

⑦ $22° 30' = 22,5° = \frac{45°}{2}$

$$\cancel{\operatorname{tg} 45°}^{1} = \frac{2 \operatorname{tg}(22,5°)}{1 - \operatorname{tg}^2(22,5°)} \quad . \quad \text{SEJA } \operatorname{tg}(22,5°) = x.$$

$$\frac{2x}{1-x^2} = 1 \Rightarrow 2x = 1 - x^2 \Rightarrow x^2 + 2x - 1 = 0$$

$\Delta = 4 - 4 \cdot 1 \cdot (-1)$

$\Delta = 8$

$$x = \frac{-2 \pm \sqrt{8}}{2} = \frac{-2 \pm 2\sqrt{2}}{2}$$

$x_1 = -1 - \sqrt{2} \simeq -2,41$

$x_2 = -1 + \sqrt{2} \simeq 0,41$

Como $22,5° \in 1°$ Quad.,

$\operatorname{tg}(22° 30') \simeq 0,41$ b

(8) $\left.\begin{matrix} tg\alpha = 6 \\ \alpha+\beta = 45^\circ \end{matrix}\right\}$ $tg\underbrace{(\alpha+\beta)}_{45^\circ} = \dfrac{tg\alpha + tg\beta}{1 - tg\alpha tg\beta}$, $tg\beta = x$

$1 = \dfrac{6+x}{1-6x} \Rightarrow 1 - 6x = 6 + x \Rightarrow 1 - 6 = x + 6x$

$\Rightarrow 7x = -5 \Rightarrow x = \dfrac{-5}{7}$

$\therefore \quad tg\beta = -5/7$

(9) $\text{sen}\, x = \dfrac{1}{3} \Rightarrow \cos^2 x = 1 - \dfrac{1}{9} = \dfrac{8}{9} \Rightarrow \cos x = \pm\dfrac{\sqrt{8}}{3} = \pm\dfrac{2\sqrt{2}}{3}$

$0 < x < \dfrac{\pi}{2}$ (1º QUAD.) $\Rightarrow \cos x = \dfrac{2\sqrt{2}}{3}$

~ $\text{sen}\left(\dfrac{\pi}{6} - x\right) = \text{sen}\dfrac{\pi}{6} \cdot \cos x - \text{sen}\, x \cdot \cos\dfrac{\pi}{6}$

(OBS: $\dfrac{\pi}{6}$ Rad $= 30^\circ$) $= \dfrac{1}{2} \cdot \dfrac{2\sqrt{2}}{3} - \dfrac{1}{3} \cdot \dfrac{\sqrt{3}}{2} = \dfrac{2\sqrt{2} - \sqrt{3}}{6}$

~ $\text{sen}(2x) = 2\,\text{sen}\, x \cos x = 2 \cdot \dfrac{1}{3} \cdot \dfrac{2\sqrt{2}}{3} = \dfrac{4\sqrt{2}}{9}$

(10) $\operatorname{tg}10^\circ\left(\underbrace{\sec 5^\circ + \operatorname{cossec}5^\circ}\right)(\cos 5^\circ - \operatorname{sen}5^\circ)$

$$\frac{1}{\cos 5^\circ} + \frac{1}{\operatorname{sen}5^\circ}$$

$$= \operatorname{tg}10^\circ \cdot \left(\cancel{1} - \frac{\operatorname{sen}5^\circ}{\cos 5^\circ} + \frac{\cos 5^\circ}{\operatorname{sen}5^\circ} - \cancel{1}\right) = \operatorname{tg}10^\circ\left(-\operatorname{tg}5^\circ + \frac{1}{\operatorname{tg}5^\circ}\right)$$

$$= \frac{2\cancel{\operatorname{tg}5^\circ}}{\cancel{1-\operatorname{tg}^2 5^\circ}}\left(\frac{\cancel{1-\operatorname{tg}^2 5^\circ}}{\cancel{\operatorname{tg}5^\circ}}\right) = 2 \quad \text{(a)}$$

$$\frac{-\operatorname{tg}^2 5^\circ + 1}{\operatorname{tg}5^\circ}$$

(11) $\frac{\operatorname{sen}80^\circ}{2}\left(\underbrace{\operatorname{tg}40^\circ + \operatorname{cotg}40^\circ}\right) = \frac{\operatorname{sen}80^\circ}{2} \cdot \frac{1}{\operatorname{sen}40^\circ \cdot \cos 40^\circ}$

EXERCÍCIO Nº 5 $\rightarrow \frac{1}{\operatorname{sen}40^\circ \cdot \cos 40^\circ}$

$$= \frac{\cancel{2}\,\cancel{\operatorname{sen}40^\circ} \cdot \cancel{\cos 40^\circ}}{\cancel{2}} \cdot \frac{1}{\cancel{\operatorname{sen}40^\circ} \cdot \cancel{\cos 40^\circ}} = 1 \quad \text{(c)}$$

(12) $\operatorname{sen}(\theta + \pi/6) + \cos(\theta + \pi/3) = \operatorname{sen}\theta \cdot \underbrace{\cos \tfrac{\pi}{6}}_{\sqrt{3}/2} + \underbrace{\operatorname{sen}\tfrac{\pi}{6}}_{1/2} \cdot \cos\theta +$

$$\cos\theta \cdot \underbrace{\cos\tfrac{\pi}{3}}_{\frac{1}{2}} - \operatorname{sen}\theta \cdot \underbrace{\operatorname{sen}\tfrac{\pi}{3}}_{\frac{\sqrt{3}}{2}} = \cancel{\frac{\sqrt{3}}{2}\operatorname{sen}\theta} + \frac{1}{2}\cos\theta + \frac{1}{2}\cos\theta - \cancel{\frac{\sqrt{3}}{2}\operatorname{sen}\theta}$$

$= \cos\theta$ (c)

(13) $(\operatorname{sen}x + \operatorname{sen}y)^2 + (\cos x + \cos y)^2$

$$\operatorname{sen}^2 x + 2\operatorname{sen}x\cos y + \operatorname{sen}^2 y + \cos^2 x + 2\cos x\cos y + \cos^2 y$$

$$1 + 1 + 2\left(\underbrace{\cos x\cos y + \operatorname{sen}x\operatorname{sen}y}_{\cos(x-y)}\right) = 2 + 2 \cdot \cos 60^\circ = 2 + \cancel{2} \cdot \frac{1}{\cancel{2}}$$

$= 2 + 1 = 3$ (d)

(14) $\text{Sen}\,x = \dfrac{1}{3} \Rightarrow \cos x = \pm \dfrac{2\sqrt{2}}{3}$

EXERCÍCIO Nº 9

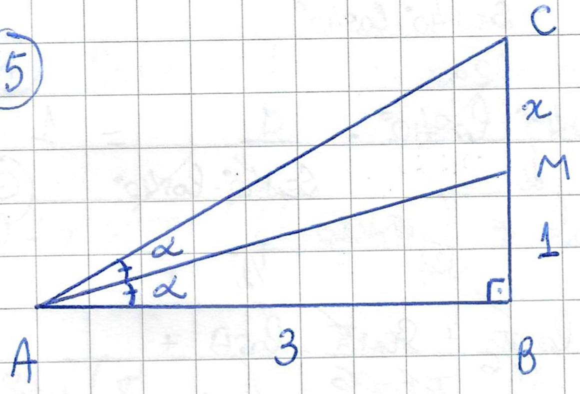
$\dfrac{\pi}{2} < x < \pi$ (2º QUAD.)

$$\cos x = -\frac{2\sqrt{2}}{3}$$

$$\text{Sen}(2x) = 2\,\text{sen}\,x \cos x = 2 \cdot \frac{1}{3} \cdot \left(-\frac{2\sqrt{2}}{3}\right) = -\frac{4\sqrt{2}}{9}$$

$$\cos(2x) = \cos^2 x - \text{Sen}^2 x = \left(-\frac{2\sqrt{2}}{3}\right)^2 - \left(\frac{1}{3}\right)^2 = \frac{8}{9} - \frac{1}{9} = \frac{7}{9}$$

(15)




$$tg\,\alpha = \frac{1}{3}$$

$$tg(2\alpha) = \frac{x+1}{3}$$

$$tg(2\alpha) = \frac{2\,tg\,\alpha}{1 - tg^2(\alpha)}$$

$$\Rightarrow \frac{x+1}{3} = \frac{2 \cdot \frac{1}{3}}{1 - \left(\frac{1}{3}\right)^2}$$

$$\Rightarrow \frac{x+1}{3} = \frac{2/3}{1 - \frac{1}{9}} \Rightarrow \frac{x+1}{3} = \frac{2/3}{8/9}$$

$$\Rightarrow \frac{x+1}{3} = \frac{\cancel{2}}{\cancel{3}} \cdot \frac{\cancel{9}\,3}{\cancel{8}\,4} \Rightarrow 4(x+1) = 3 \cdot 3$$

$$\Rightarrow 4x + 4 = 9 \Rightarrow 4x = 5 \therefore x = 5/4 = 1{,}25\text{ m}$$

(b)

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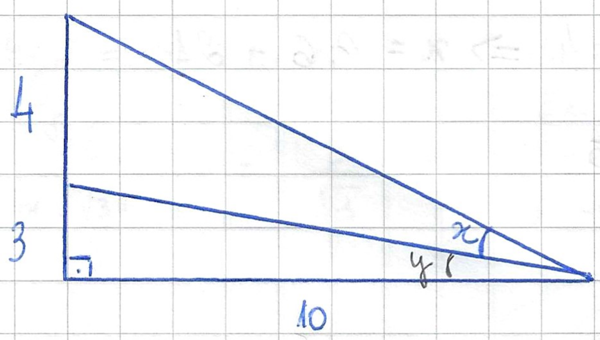


$$tg\,y = \frac{3}{10}, \quad tg(x+y) = \frac{7}{10}; \quad tg(x+y) = \frac{tg\,x + tg\,y}{1 - tg\,x\,tg\,y}$$

SEJA $tg\,x = a$ ENTÃO,

$$\frac{a + 3/10}{1 - a \cdot \frac{3}{10}} = \frac{7}{10} \Rightarrow \frac{\frac{10a+3}{\cancel{10}}}{\frac{10 - 3a}{\cancel{10}}} = \frac{7}{10}$$

$$\Rightarrow \frac{10a+3}{10-3a} = \frac{7}{10} \Rightarrow 10(10a+3) = 7(10-3a)$$

$$\Rightarrow 100a + 30 = 70 - 21a$$

$$\Rightarrow 121a = 40$$

$$\Rightarrow a = \frac{40}{121} \qquad \therefore \boxed{tg\,x = \frac{40}{121}}$$

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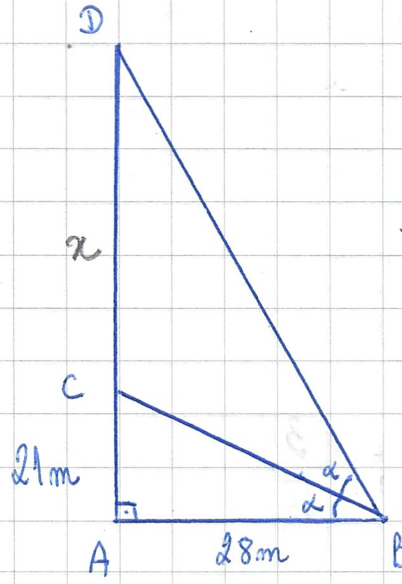



$$tg\,\alpha = \frac{\cancel{21}}{\cancel{28}} = \frac{3}{4}$$

$$tg(2\alpha) = \frac{x+21}{28} = \frac{2\,tg\,\alpha}{1 - tg^2\alpha}$$

$$\Rightarrow \frac{x+21}{28} = \frac{2 \cdot 3/4}{1 - (3/4)^2} \Rightarrow \frac{x+21}{28} = \frac{3/2}{1 - \frac{9}{16}}$$

$$\Rightarrow \frac{x+21}{28} = \frac{3/2}{7/16} \Rightarrow \frac{x+21}{28} = \frac{3}{\cancel{2}} \cdot \frac{\overset{8}{\cancel{16}}}{7} \Rightarrow$$

$$\frac{x+21}{\cancel{28}\;4} = \frac{24}{\cancel{7}} \Rightarrow x+21 = 24\cdot 4 \Rightarrow x = 96-21$$

$$\Rightarrow x = 75$$

$\therefore\ \overline{AD} = 75 + 21 = 96\,m$

(18) $\left(\cos 15^\circ + \operatorname{sen} 15^\circ\right)^2$

$$= \underbrace{\cos^2 15^\circ + 2\operatorname{sen}15^\circ\cdot\cos 15^\circ + \operatorname{sen}^2 15^\circ}_{1 \text{ (}\cos^2 15^\circ + \operatorname{sen}^2 15^\circ\text{)}} = 1 + \underbrace{2\operatorname{sen}15^\circ\cdot\cos 15^\circ}_{\operatorname{sen}30^\circ}$$

$$= 1 + \frac{1}{2} = \frac{3}{2}$$

(19)

EXERCÍCIO Nº 5

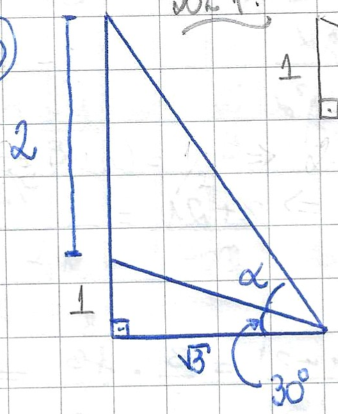
$$\operatorname{tg}x + \operatorname{cotg}x = \frac{1}{\operatorname{sen}x\cos x}$$

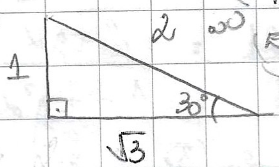
$$\operatorname{sen}(2x) = 2\operatorname{sen}x\cdot\cos x = \frac{1}{2} \Rightarrow \operatorname{sen}x\cos x = \frac{1}{4}$$

$$\Rightarrow \operatorname{tg}x + \operatorname{cotg}x = \frac{1}{\frac{1}{4}} = 1\cdot\frac{4}{1} = 4 \quad \text{(C)}$$

(20)

SOL 1:




TRIÂNGULO EGÍPCIO

$$\operatorname{tg}(\alpha + 30^\circ) = \frac{\operatorname{tg}\alpha + \operatorname{tg}30^\circ}{1 - \operatorname{tg}\alpha\,\operatorname{tg}30^\circ} = \frac{3}{\sqrt{3}}$$

Seja $tg\,\alpha = x$. Então:

$$\frac{x + \frac{\sqrt{3}}{3}}{1 - x\cdot\frac{\sqrt{3}}{3}} = \frac{3}{\sqrt{3}} \Rightarrow \frac{\frac{3x+\sqrt{3}}{\cancel{3}}}{\frac{3 - x\sqrt{3}}{\cancel{3}}} = \frac{3}{\sqrt{3}}$$

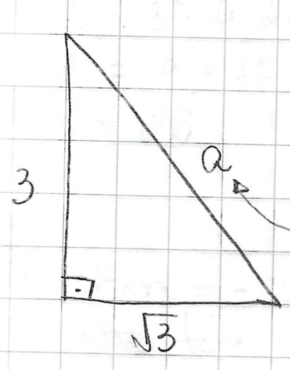
$$\Rightarrow \frac{3x+\sqrt{3}}{3-x\sqrt{3}} = \frac{3}{\sqrt{3}} \Rightarrow (3x+\sqrt{3})\sqrt{3} = 3(3-x\sqrt{3})$$

$$\Rightarrow 3\sqrt{3}x + 3 = 9 - 3\sqrt{3}x \Rightarrow 3\sqrt{3}x + 3\sqrt{3}x = 9 - 3$$

$$\Rightarrow 6\sqrt{3}x = 6 \Rightarrow x = \frac{\cancel{6}}{\cancel{6}\sqrt{3}} = \frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} \Rightarrow x = \frac{\sqrt{3}}{3}$$

$$\therefore\ tg\,\alpha = \frac{\sqrt{3}}{3}$$

Sol 2:



$$a^2 = 3^2 + (\sqrt{3})^2$$
$$a^2 = 9 + 3$$
$$a^2 = 12$$
$$a = \sqrt{12} \Rightarrow a = 2\sqrt{3}$$

Como a hipotenusa $(2\sqrt{3})$ é o dobro do cateto $(\sqrt{3})$ o triângulo também é egípcio.

$$\alpha + 30^\circ = 60^\circ \Rightarrow \alpha = 30^\circ$$

$$tg\,\alpha = tg\,30^\circ = \frac{\sqrt{3}}{3}$$